Precalculus Assignment Sheet Unit 10

Problems which Require the Use of all Math Skills

Assignment 1	2.3 Average Rate of Change HW: p. 80 # 57, 59, 69, 71a, c, 77, 79
Assignment 2	2.4 Piecewise Defined & Absolute Value Functions HW: Complete Worksheet & p. 89 #33, 35, 37, 43, 47
Assignment 3	2.6 Problem Solving with Geometry HW: Complete Worksheet & p. 107 # 11, 13, 15, 24
Assignment 4	2.6 Problem Solving with Graphing HW: Complete Worksheet & p. 107 # 1-9 odd
Assignment 5	11.1 Systems of Linear Equations HW: Complete Worksheet & p. 700 #43 p. 716 # 75
Assignment 6	Math for College Courses HW: Complete Worksheet
Assignment 7	Appendix Algebra & Calculator Skills: HW: Complete Worksheet & p. A34 #7, 17; p. A89 #53, 89, 101
Assignment 8	14.1 Limits HW: p. 860 # 7, 9, 17-21 odd, 29, 31, 37, 39, 41
Assignment 9	14.3 Continuity HW: p. 873 # 7-31 odd
Review	Complete Supplementary Worksheet

2.3 Average Rate of Change

HW: p. 80: 55, 57, 59, 69, 71, 77, 79

1. Find the average rate of change in $f(x) = x^2 + 12$ from $x = -1$ to $x = 4$.

2. Let $f(x) = 2x^2 + 3x$. Find the slope of the secant line (average rate of change) through the points, $(x, f(x))$ and $(x+\Delta x, f(x+\Delta x))$

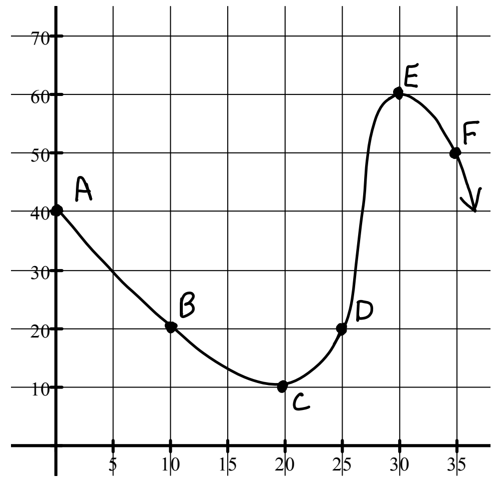
3. A stone is thrown upward from a height of 2 meters with an initial velocity of 8 meters per second. If only the effect of gravity is considered, then the stone's height in meters after t seconds is given by the equation $H(t) = -4.9t^2 + 8t + 2$.

a. Find a formula for the average velocity from 1 to $(1+\Delta t)$.

b. Use your answer in part a to find the average velocity from $t = 1$ to $t = 3.5$.

4. Refer to the graph of g at the right. Find the average rate of change in g over each interval.

a. C to E



b. $0 \leq x \leq 35$

Over what interval does the average rate of change in g have the given value?

c. 0

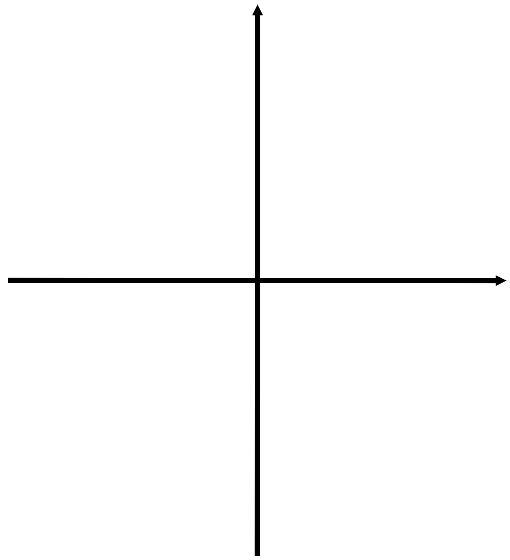
d. $-\frac{3}{2}$

5. Suppose $P = (3, 7)$ and $Q = (9, a)$ are points on the graph of the function h. If the average rate of change in h from $x = 3$ to $x = 9$ is $-\frac{5}{3}$, find a.

2.4 Piecewise Defined Functions

HW: Complete Worksheet & p. 89 #33, 35, 37, 43, 47

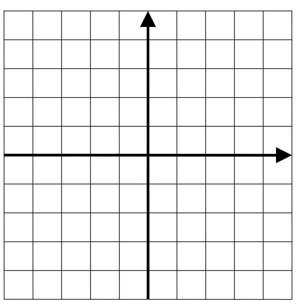
Example: Sketch: $f(x)=\begin{cases} x^2-2x-8, & x\le 1 \\ 16-x^2, & x>1 \end{cases}$



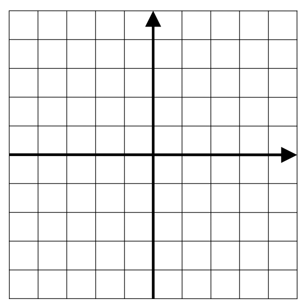
Absolute Value Functions

Write as piecewise functions and sketch a graph. Verify your graph using a calculator.

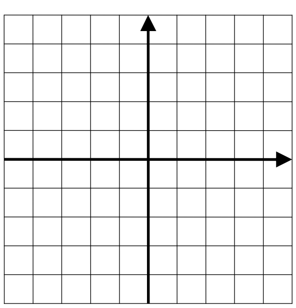
1) y = $\sqrt{x^2} = |x|$



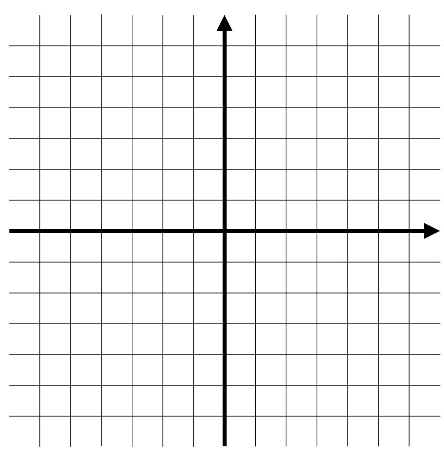
2) $y = |x-3|$



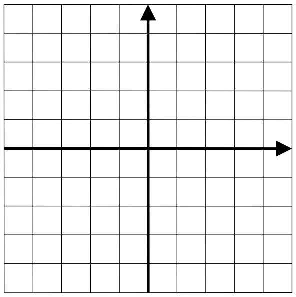
3) y = $|x+3|-|x-1|$



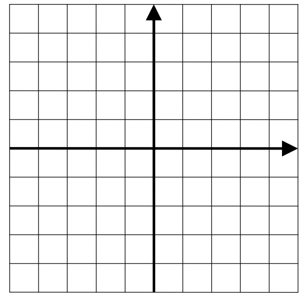
4) y = $|x|+|2x-5|$



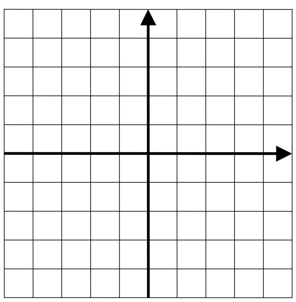
5) $y = \left|x^2 + x - 12\right|$



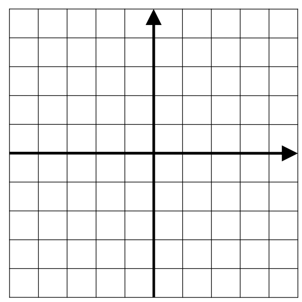
6) $y = \dfrac{-x}{|x|}$



7) $y = x|x + 4|$

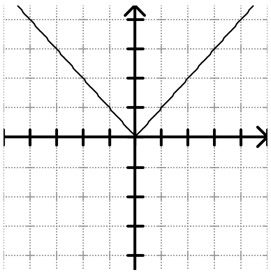


8) $y = \dfrac{\left|x^2 - 2x\right|}{x - 2}$

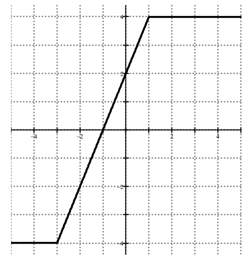


answers

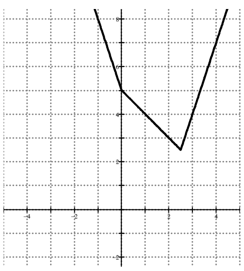
1. $y = \begin{cases} x; & x \ge 0 \\ -x; & x < 0 \end{cases}$



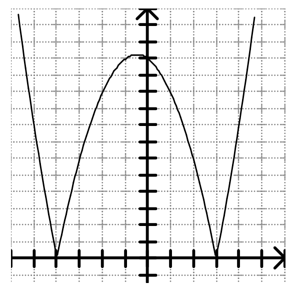
3. $y = \begin{cases} -4; & x \le -3 \\ 2x+2; & -3 < x < 1 \\ 4; & x \ge 1 \end{cases}$



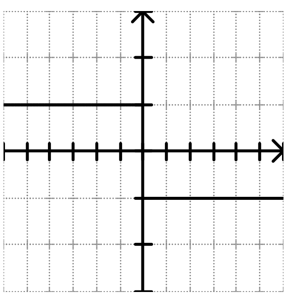
4. $y = \begin{cases} -3x+5; & x \le 0 \\ -x+5; & 0 < x < \frac{5}{2} \\ 3x-5; & x \ge \frac{5}{2} \end{cases}$



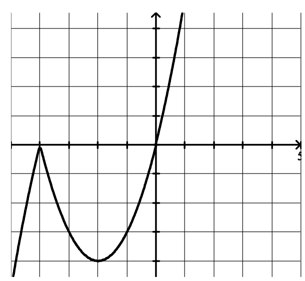
5. $y = \begin{cases} x^2+x-12; & x < -4, x > 3 \\ -\left(x^2+x-12\right); & -4 \le x \le 3 \end{cases}$



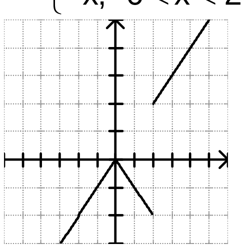
6. $y = \begin{cases} -1; & x > 0 \\ 1; & x < 0 \end{cases}$



7. $y = \begin{cases} x\left(x+4\right); & x > -4 \\ -x\left(x+4\right); & x \le -4 \end{cases}$



8) $y = \begin{cases} x; & x \le 0, x > 2 \\ -x; & 0 < x < 2 \end{cases}$



2.6 Problem Solving with Geometry
HW: Complete Worksheet & p. 107 # 11, 13, 15, 24

- Your work must be completed on a separate sheet of ***GRAPH PAPER.***
- Define all variables and draw a diagram for each problem to receive credit.

1) A container with a square base, vertical sides, and open top is to be made from 1000 ft^2 of material. Sketch and label a diagram. Write an equation for the volume of the container as a function of the length of a side of the base.

2) A right circular cylinder is inscribed in a cone with radius 6 inches and height 10 inches. Write an equation for the volume of the cylinder as a function of the radius of the cylinder.

3) A 17 ft ladder is leaning against a wall. If the bottom of the ladder is x ft from the wall and the top of the ladder is y feet above the ground, write an equation for the horizontal distance the ladder is from the bottom of the wall as a function of the vertical distance the ladder is above the ground.

4) Wheat is poured through a chute and falls in a conical pile whose radius is always half the height of the pile.

a) Write an equation for the volume of the pile as a function of the height of the pile.
b) Write an equation for the volume of the pile as a function of the radius of the pile.

Answers: **1.** $V(x) = 250x - \frac{x^3}{4}, 0 \le x \le 10\sqrt{10}$ **2.** $V(r) = 10\pi r^2 - \frac{5\pi r^3}{3}, 0 \le x \le 6$

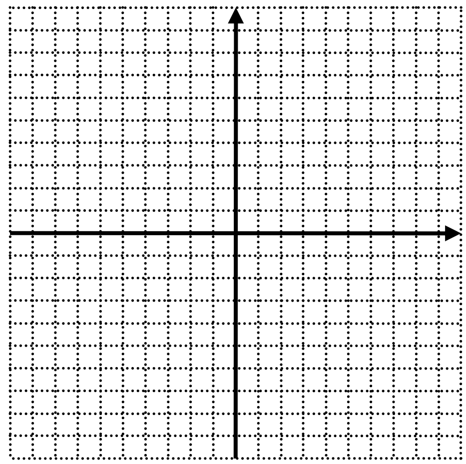
3. $x(y) = \sqrt{17^2 - y^2}$ **4a)** $V(h) = \frac{\pi h^3}{12}, h \ge 0$ **4b)** $V(r) = \frac{2\pi}{3} r^3, r \ge 0$

2.6 Problem Solving with Graphs

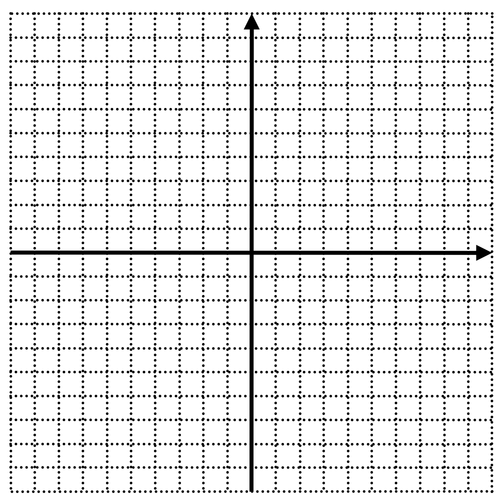
HW: Complete Worksheet & p. 107 # 1-9 odd

- Your work must be completed on a separate sheet of ***GRAPH PAPER.***
- Define all variables and draw a diagram for each problem to receive credit.

1) Write the equation for the distance between the point $(0,9)$ and the curve $x = 2y^2$ as a function of y.

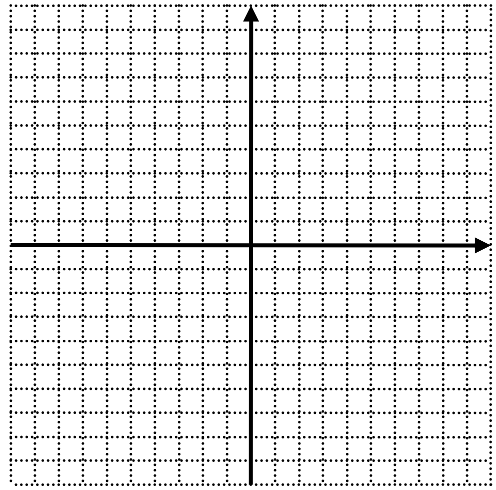


2) A rectangle has its two lower corners on the x-axis and its two upper corners on the curve $y = 16 - x^2$. Write an equation of the area of the rectangle as a function of y.



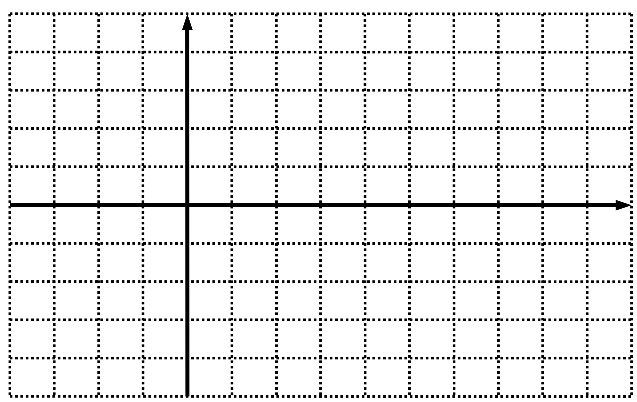
3) Sketch the functions, shade the region enclosed by the functions, and label all points of intersection on the boundary of the functions.

$y = x^2$ *and* $y = x + 6$



4) Sketch the functions, shade the region enclosed by the functions, and label all points of intersection on the boundary of the functions.

$y = \cos 2x,\ \ y = 0,\ \ x = \frac{\pi}{4},$ *and* $x = \frac{\pi}{2}$



Answers:

1) $d(y) = \sqrt{4y^4 + y^2 - 18y + 81}$ **2)** $A(y) = 2y\sqrt{16 - y}$ **(assuming that the rectangle is symmetric with the y axis.)** **3) points of intersection are:** $(-2,4)$ and $(3,9)$ **4) points:** $(\frac{\pi}{4},0),(\frac{\pi}{2},-1),(\frac{\pi}{2},0)$

11.1 Multivariable Linear Equations

HW: Complete Worksheet & p. 700 #43 p. 716 # 75

In Algebra II you learned the method of substitution and elimination to solve systems of equations, linear and non-linear. In this section we are going to solve systems of up to three equations using 3 variables. To review simple substitution and elimination problems, take a look at 7.1 and 7.2. These sections should familiar to you. Also remember that an equation in the form $Ax + By + Cz = D$ is a line in three dimensional space. Three lines in 3D space can have no solution, one solution or infinitely many solutions.

Ex. 1: Solve the system of linear equations using "back substitution"

$$\begin{cases} x - 2y + 3z = 9 \\ y + 3z = 5 \\ z = 2 \end{cases}$$

List the solution as a coordinate triple (, ,)

Ex. 2: Solve the system of linear equations using any method. (Take some notes on your favorite method as we go over the possibilities!)

$$\begin{cases} x - 2y + 3z = 9 \\ -x + 3y = -4 \\ 2x - 5y + 5z = 17 \end{cases}$$

List the solution as a coordinate triple (, ,) Ex. 3: The height at time *t* of an object that is moving in a (vertical) line with constant acceleration *a* is given by the **position equation**

$$s = \frac{1}{2}at^2 + v_\circ t + s_\circ$$

The height, *s*, is measured in feet, the acceleration *a* is measured in feet per second squared, *t* is measured in seconds, $v_\circ$ is the initial velocity (at $t = 0$), and $s_\circ$ is the initial height. Find the values of *a*, $v_\circ$, and $s_\circ$ if $s = 52$ at $t = 1$, $s = 52$ at $t = 2$, and $s = 20$ at $t = 3$.

Ex. 4: Find a quadratic equation, $y = ax^2 + bx + c$, whose graph passes through the points $(-1, 3), (1, 1)$, and $(2, 6)$.

Ex. 5: (Calculator) An inheritance of $12,000 was invested among three funds: a money market fun that paid 5% annually, municipal bonds that paid 6% annually, and mutual funds that paid 12% annually. The amount invested in mutual funds was $4000 more than the amount invested in municipal bonds. The total interest earned during the first year was $1120. How much was invested in each type of fund?

Math For College Courses
HW: Complete Worksheet

1. Solve for x:

$$25x^2\left(4+x^2\right)^{-\frac{3}{2}}+6x\left(4+x^2\right)^{-\frac{1}{2}}=0$$

2. Write as a piecewise function and sketch:

$$f\left(x\right)=\left|x^2-4x-5\right|$$

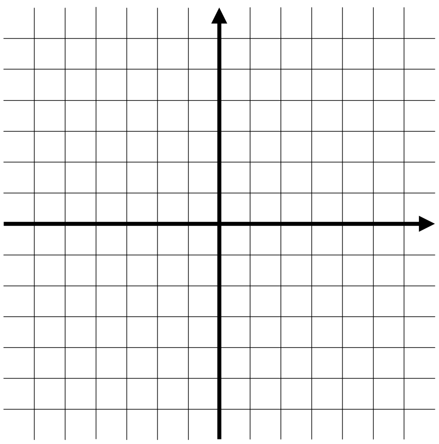
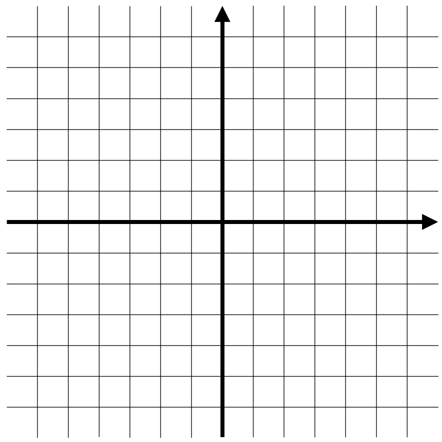
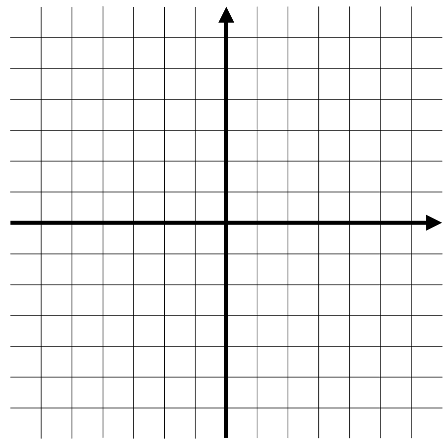
3. Simplify $\dfrac{\dfrac{1}{\sqrt{x}}-\dfrac{1}{\sqrt{w}}}{x-w}$

4. Sketch the following, without using your calculator, and identify domain and range:

$y = \sin^{-1} x$

$y = x^{\frac{2}{3}}$

$y = \dfrac{x}{|x|}$

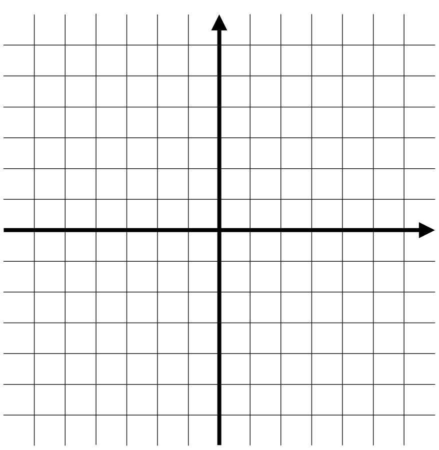
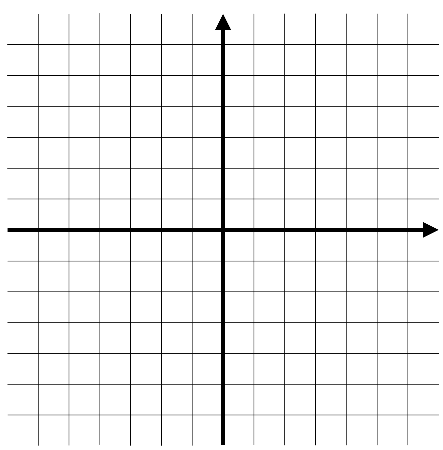
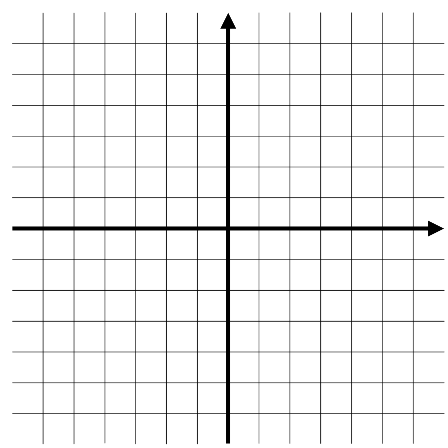




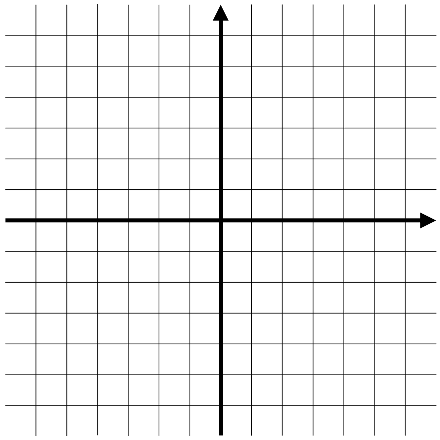
5. Sketch the following, without using your calculator, and identify the domain, range and all asymptotes.

a. $f(x) = \dfrac{(x+4)^2(x-2)}{(x+6)(x-2)}$ b. $f(x) = -(x+4)^3(x-2)^2(x+1)$

*c. $f(x) = \ln|x+1|$ *d. $f(x) = e^{x^2} + 1$

(These are more challenging)



6. Sketch the angle, identify a reference angle and find the exact value: $\sin \frac{2\pi}{3}$

7. Given that $\sin x = \frac{3}{5}$, $0 < x < \frac{\pi}{2}$, find $\sin 2x$

8. Solve $\sin^2 x - \cos^2 x = 0$ on $[0, 2\pi)$

9. Solve the equation without using your calculator: $y^3 - 2y + 1 = 0$.

10. Solve

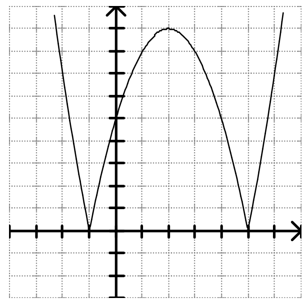
a. $\ln\left(x^2+4\right) - \ln\left(x+2\right) = 2 + \ln\left(x-2\right)$

b. $5^x + 125 \cdot 5^{-x} = 30$

Math for College Courses -- Answers

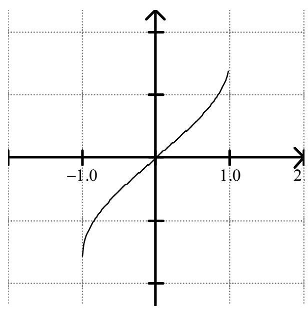
1. $x = 0, -\frac{8}{3}, -\frac{3}{2}$

2. $f(x) = \begin{cases} x^2 - 4x - 5, & x \le -1 \text{ or } x \ge 5 \\ -x^2 + 4x + 5, & -1 < x < 5 \end{cases}$



3. $-\frac{1}{\sqrt{wx}\left(\sqrt{w} + \sqrt{x}\right)}$

4.



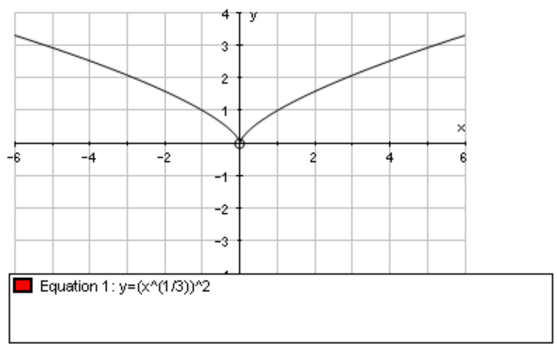
$y = \sin^{-1} x$

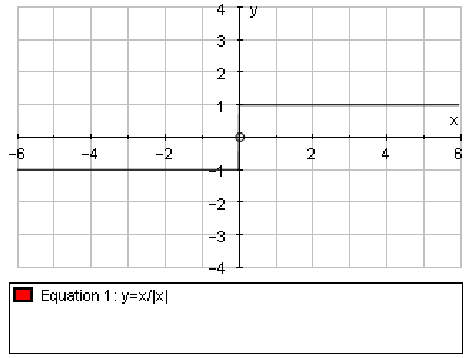
D: [-1,1], R: $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

$y = x^{\frac{2}{3}}$

Domain: $(-\infty, \infty)$

Range: $[0, \infty)$





$y = \frac{x}{|x|}$

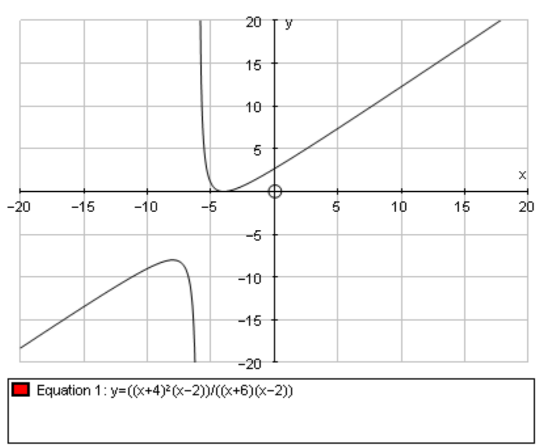
Domain: $(-\infty, 0) \cup (0, \infty)$

Range: $\{-1, 1\}$

5. Sketch the following, without using your calculator, and identify the domain, range and all asymptotes.

a. $f(x) = \frac{(x+4)^2(x-2)}{(x+6)(x-2)}$

b. $f(x) = -(x+4)^3(x-2)^2(x+1)$



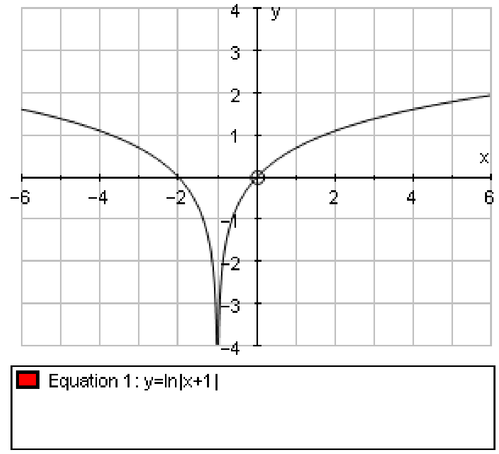
*c. $f(x) = \ln|x+1|$

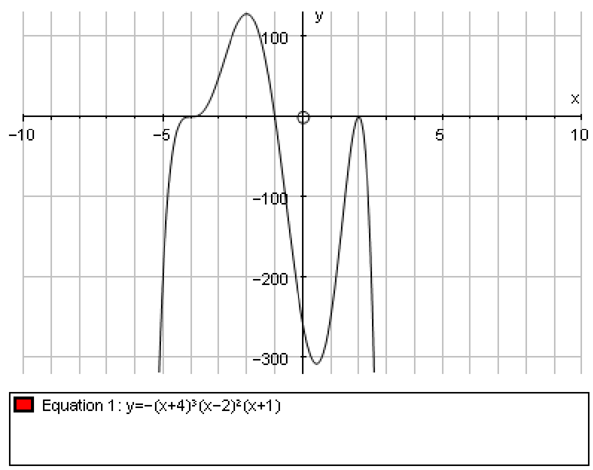
*d. $f(x) = e^{x^2} + 1$

Domain: all reals except x = -6, 2

Range: $(-\infty, -8] \cup [0, \infty)$

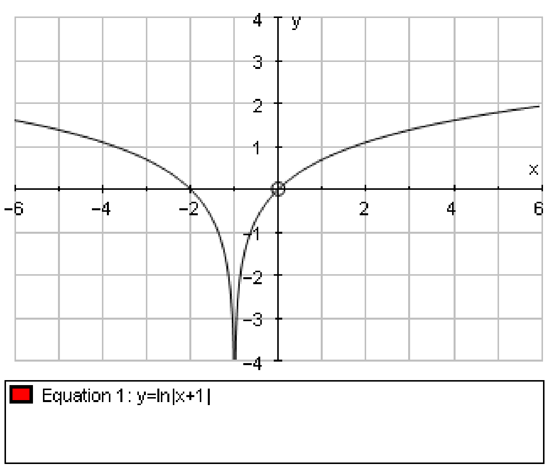
Asymptotes: x= -6, y= x + 2
D





Domain: $(-\infty, \infty)$

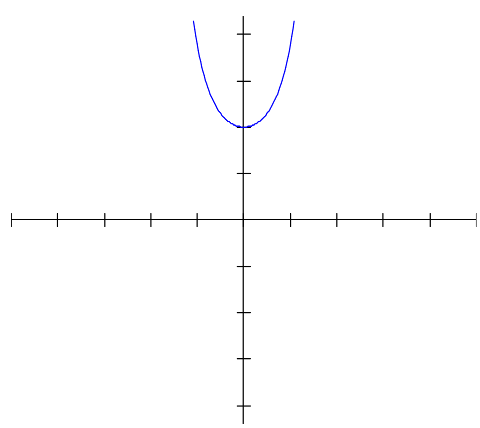
Range: $(-\infty, 128]$



Domain: all reals except x = -1

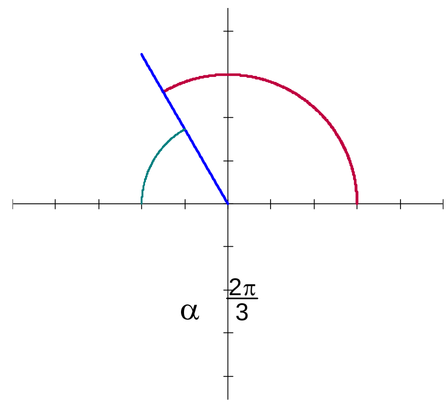
Range: $(-\infty, \infty)$

Asymptotes: $x = -1$



$f(x) = e^{x^2} + 1$ Domain: $(-\infty, \infty)$ Range: $[2, \infty)$

6. α is the reference angle, $\alpha = \frac{\pi}{3}$ $\sin\frac{2\pi}{3} = \frac{\sqrt{3}}{2}$



7. sin 2x = 24/25

8. $x = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

9. $y = 1, \frac{-1 \pm \sqrt{5}}{2}$

10. a. $x = 2\sqrt{\frac{e^2+1}{e^2-1}}$ b. x = 1, 2

Algebra & Calculator Worksheet for Pre-Calculus
HW: Complete Worksheet & p. A34 #7, 17; p. A89 #53, 89, 101

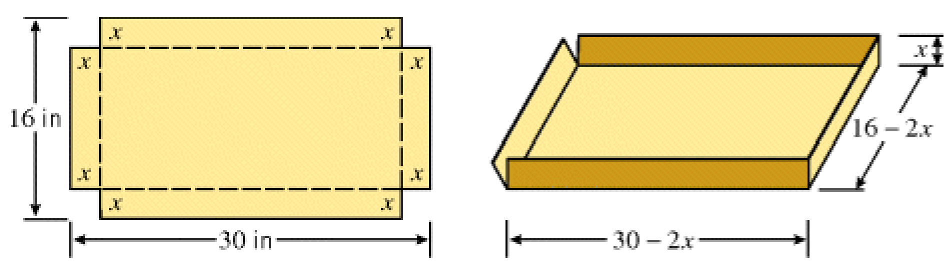
1) Simplify the expression:

$$\frac{2x(1-x^2)^{1/3}+\frac{2}{3}x^3(1-x^2)^{-2/3}}{(1-x^2)^{2/3}} \qquad x \neq 1,-1$$

2) Write a simplified function that is identical to the original function. $f(x)=\frac{x^3+1}{x^2-1}$

3) Sketch $f(x)=x^2+1$ and the point (3,0). Write the function d(x) which expresses the distance between an arbitrary point on f(x) and (3,0). Use your calculator and d(x) to find the shortest distance from (3,0) to the graph. What is this distance? What are the coordinates of the point on the graph where this occurs?

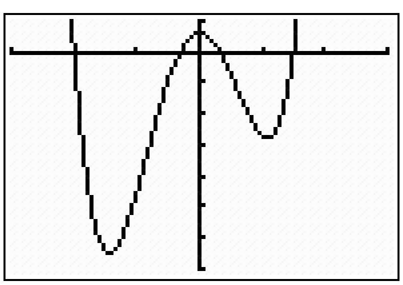
4) An open box is to be made from a 16-inch by 30-inch piece of cardboard by cutting out squares of equal size from the four corners and bending up the sides. Write a function, V(x), for the volume of the box (include its domain). Use your calculator to find the maximum volume. What size should the squares be to obtain this box?



Day 1: Calculator Worksheet

1. Sketch the function, $f(x) = x^4 + 5x^3 - 310x^2 - 50x + 3000$, accurately displaying its characteristics.
2. Find all zeros of the polynomial $p(x) = x^4 - 2x^3 - 5x^2 + 4x + 6$. Using synthetic division, **verify** all rational roots.
3. Find all real solutions to the equation: $-141.12 + 21k + k^3 = -149$
4. Find the point of intersection in the first quadrant for $y = e^{-x^2}$ and $y = 1 - \cos x$
5. Solve the inequality: $80 - 10\cos\left(\frac{\pi t}{12}\right) \geq 78$ on the interval [0, 24).
6. Find the following function values to 3 decimal places:
   a. $f(x) = \frac{x^3}{e^x}, f(5), f(10), f(20)$
   b. $f(x) = (1+x)^{1/x}, f(1), f(.1), f(.01)$

Answers: 1. $\frac{2x(3-2x^2)}{3(1-x^2)^{4/3}}$ 2. $f(x) = \frac{x^2 - x + 1}{x - 1}, x \neq -1$ 3. Min dist: 2.74, Coord: (.735, 1.540) 4. Max Vol:726 cu in Squares: 3 1/3 in X 3 1/3 in

1.  2. $\{-1, 3, \pm\sqrt{2}\}$ 3. -.373 4. (0.942, 0.412) 5. $5.230 \leq t \leq 18.769$

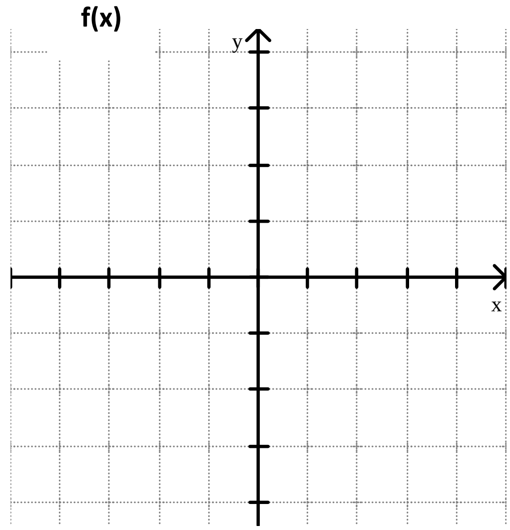
6. a. 0.842, 0.045, 0.000 b. 2, 2.594, 2.705

14.1 Limits—A Graphical Approach

HW: p. 860 # 7, 9, 17-21 odd, 29, 31, 37, 39, 41

$f(x)=\dfrac{x^2-4}{x-2}, x \neq 2$.

Describe the curve at x = 2.



Describe the behavior of the y-values near x = 2.

Use your calculator to find the y – values (4 decimal place accuracy) near x = 2.

X	1.75	1.9	1.99	1.999	2	2.001	2.01	2.1	2.25
f(x)									

From this table, as x approaches 2, the value of f(x) approaches ______________.

In calculus, we say that the limit of f(x) as x approaches 2 is ___________. Or $\lim_{x\to 2} f(x) = \lim_{x\to 2}\dfrac{x^2-4}{x-2} =$ _________..

We can work with more complicated situations to describe how a curve behaves if we introduce right and left hand limits, we we denote: $\lim_{x\to c^+} f(x)$ and $\lim_{x\to c^-} f(x)$.

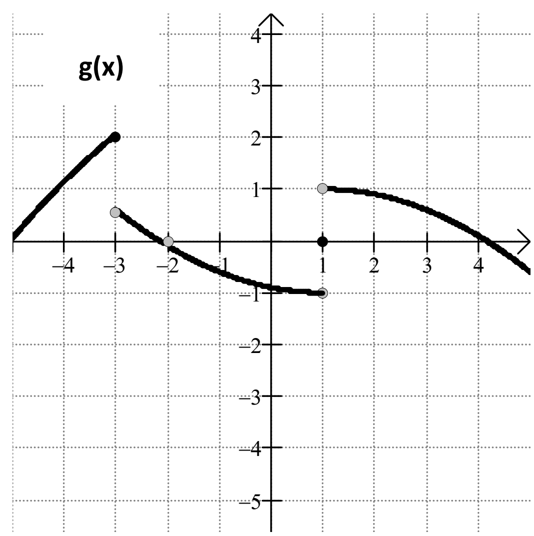
Example: For g(x), shown in the graph, find

a. $\lim_{x\to -3^-} g(x)$ b. $\lim_{x\to -3^+} g(x)$ c. $\lim_{x\to -3} g(x)$

d. $\lim_{x\to -2^+} g(x)$ e. $\lim_{x\to -2^-} g(x)$ f. $\lim_{x\to -2} g(x)$

g. $\lim_{x\to 1^+} g(x)$ h. $\lim_{x\to 1^-} g(x)$ i. $\lim_{x\to 1} g(x)$

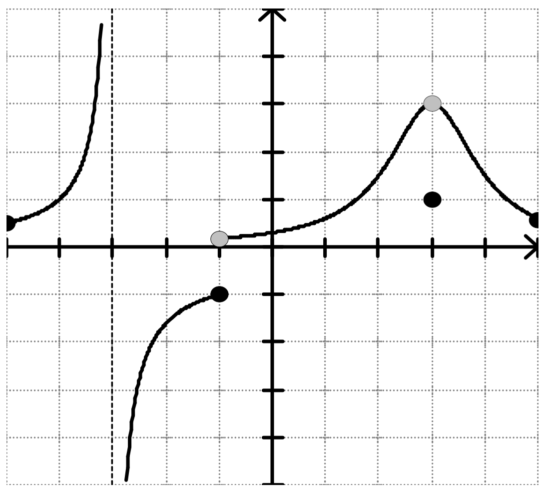
j. $\lim_{x\to 4^+} g(x)$ k. $\lim_{x\to 4^-} g(x)$ l. $\lim_{x\to 4} g(x)$



14.3 Continuity

HW: p. 873 # 7-31 odd

Definition	The function, f, is continuous at x = c, provided
	1. f(c) is defined
	2. $\lim_{x \to c} f(x)$ exists
	3. $\lim_{x \to c} f(x) = f(c)$



Explain why the function is not continuous for the following values of x by referring to the part of the definition which is violated.

a. x = 3

b. x = -1

c. x = -3

Use the definition to verify that the function is continuous at:

a. x = 0

b. x = -1

c. x = 4

Precalculus Name ___________________________________

<u>Review</u>

Please fully demonstrate reasoning. Provide a diagram, sketch or definition of variables when appropriate. Label any graphs with appropriate units.

1. A piece of wire πy inches long could be bent into a circle or a square.
   a) If the wire is bent into a circle, what is the area of that circle in terms of ***y***?

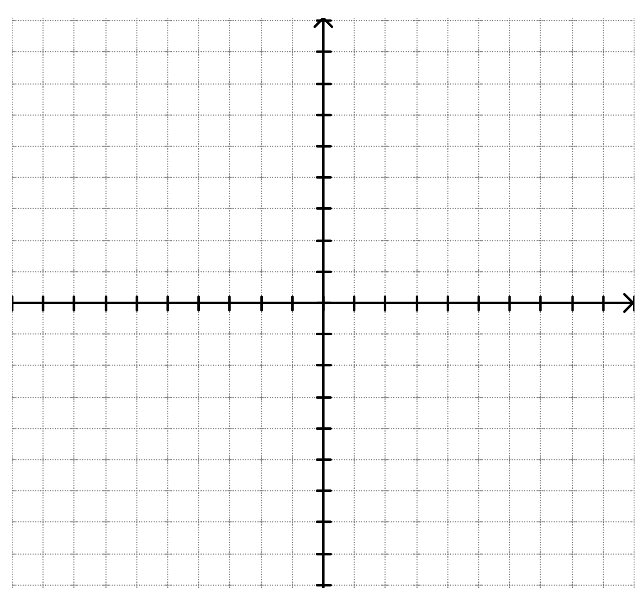
   b) If the wire is bent into a square, what is the area of that square in terms of ***y***?

   c) Which area is larger? Why?

2. $f(x) = |x-1|, g(x) = 2 - \ln x$

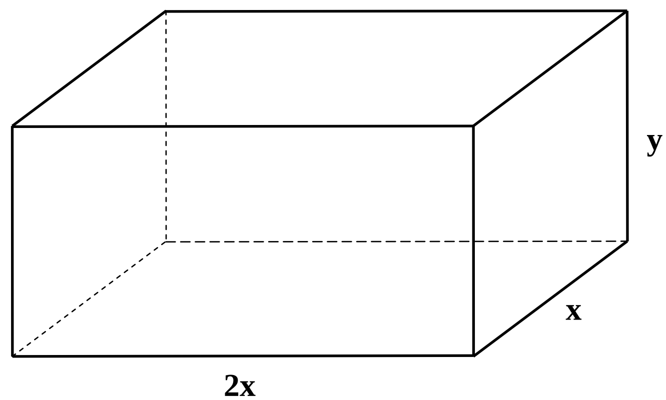
   a. Find the domain and range of $(f \circ g)(x)$

b. Sketch $(f \circ g)(x)$, labeling asymptotes and intercepts.



3. A line with slope m $(m < 0)$ passes through a point (a, b) (a and b are constants) in the first quadrant. Express the area of the triangle bounded by this line, the x-axis and y-axis in terms of ***a, b, and m.***

4. A 36 inch wire is cut into 12 pieces which are soldered (joined) together to form a rectangular frame whose base is twice as long as it is wide. The frame is then covered with paper forming a box.



a) State an expression for the volume $[V(x)]$ of the box in terms of x, with domain.

b) What are the dimensions of the box with maximum volume? (Hint: Calculator)

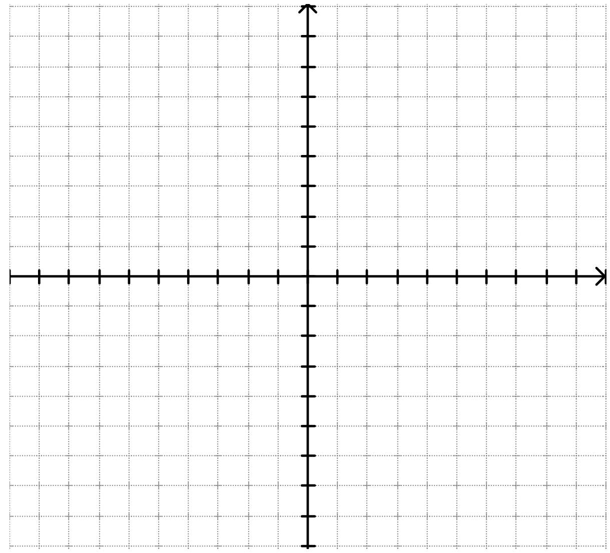
5. $f(x) = \left|\frac{1}{2} - \cos(2x)\right|, [0, \pi]$

a. Find the zeros of f(x)

b. Rewrite f(x) as a piecewise function

6. Sketch the functions, shade the region enclosed by the functions, and label all points of intersection on the boundary of the functions.

$y = \arctan x, \quad y = \frac{\pi}{4}$ and the *y-axis*



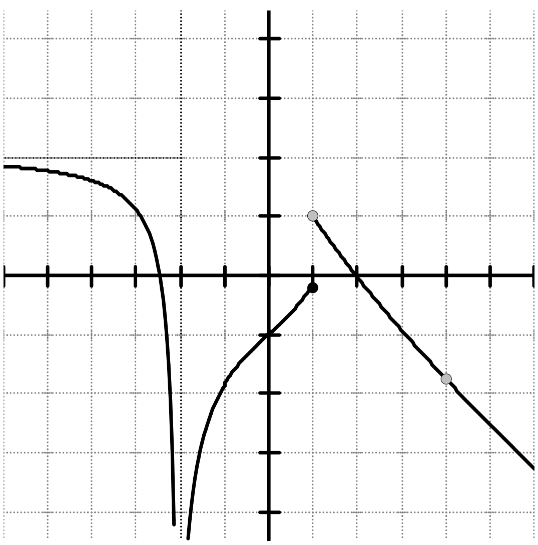
7. Evaluate the following limits or function values, based on the graph of *f(x)* below. If a limit does not exist, explain why.

a. $\lim_{x \to -2^-} f(x)$

b. $\lim_{x \to -2^+} f(x)$

c. $\lim_{x \to -2} f(x)$

d. $f(-2)$



e. $\lim_{x \to -3^-} f(x)$

f. $\lim_{x \to -3^+} f(x)$

g. $\lim_{x \to -3} f(x)$

h. $f(-3)$

i. $\lim_{x \to 1^-} f(x)$

j. $\lim_{x \to 1^+} f(x)$

k. $\lim_{x \to 1} f(x)$

l. $f(1)$

m. $\lim_{x \to 4^+} f(x)$

n. $\lim_{x \to 4^-} f(x)$

p. $\lim_{x \to 4} f(x)$

q. $f(4)$

r. $\lim_{x \to -\infty} f(x)$

s. $\lim_{x \to \infty} f(x)$

8. State the values of x where the above function is discontinuous and justify