## Precalculus Take Home problem

## Law of Sines/Cosines

- This problem is due at the beginning of class on $\qquad$ .
- While you may work with other students in the Precalculus class, you should write up your solutions on your own. If you do work with other students please indicate that in your work. You are NOT allowed to consult any other adults except your classroom teacher. This means that tutors / private teachers cannot assist you. You are allowed to use your textbook, notes and handouts from class to help you.
- On each part, you will be graded not only on your accuracy/answer, but also on the path you take to a final answer. See the rubric on the back of this page. YOU ARE RESPONSIBLE FOR CLEARLY LABELING EACH PART SO THAT WE CAN FIND YOUR WORK.

In Japan, the Nagoya Airport and Tokyo Airports are 260 km apart. (Nagoya is roughly due west of Tokyo.) At the Tokyo airport, there's a radar monitor which can keep track of all planes within a $100-\mathrm{km}$ radius of the airport.

1. Plane \#1 is 220 km from Nagoya airport, and is flying 32 degrees from the straight line between the two airports. Is the plane within range of the radar at Tokyo airport? Show a clear diagram and path to your answer.
2. Plane \#2 is going to take off from Nagoya Airport and fly past Tokyo Airport. Its path will make an angle of $\theta$ with the line between the airports. If $\theta=15^{\circ}$, how far will Plane \#2 be from Nagoya airport when it first comes within range of the Tokyo radar? How far away will it be from Nagoya when it is last within range? Store both of these distances as P and Q on your calculator, without rounding.

3a. Suppose that plane \#3 takes off from Nagoya airport, and makes an angle of $\theta=40^{\circ}$ with the line between the airports. Draw a picture in which "x" represents the distance from plane \#3 to Nagoya, and " y " represents the distance from plane \#3 to Tokyo. Express the distance "y" in terms of "x."

3 b . Use your equation in part (3a) to show that in the case where $\theta=40^{\circ}$, Plane \#3 is never in the range of the Tokyo Radar.

3c. For which value of $\theta$ will a plane from Nagoya airport be within the range of the Tokyo Radar at exactly one point. Find the distance between the plane and Nagoya and store this distance in your calculator as " F ", without rounding.
4. Show numerically that the square of "F" in part (3c) is exactly equal to the product of the distances " P " and "Q" in part (2). What theorem in geometry expresses this fact?

GRADING RUBRIC: Staple this to the front of your work.
Name $\qquad$
I worked with: $\qquad$

| Criterion | Possible | Score |
| :---: | :---: | :---: |
| Part (1) 5 total |  |  |
| - Diagram showing appropriate labels. | 2 |  |
| - Equation leading to final answer. | 2 |  |
| - Accuracy/answer | 1 |  |
|  |  |  |
| Part (2) 5 total |  |  |
| - Diagram | 2 |  |
| - Equation allowing for students to get the right answer | 2 |  |
| - Correct algebra accuracy to final answer | 1 |  |
|  |  |  |
| Part (3a) 3 total |  |  |
| - Diagram with required features | 1 |  |
| - Gets y as expression in terms of $x$, with accurate algebra | 2 |  |
|  |  |  |
| Part (3b) 3 total |  |  |
| - Work reflects the key goal needed to solve the problem | 1 |  |
| - Reasoning to the correct conclusion provided, path is clear. | 2 |  |
|  |  |  |
| Part (3c) 4 total |  |  |
| - Diagram is detailed enough to show the goal of the problem. | 2 |  |
| - Setup to final answer correct | 1 |  |
| - Correct algebra/arithmetic to final answer | 1 |  |
|  |  |  |
| Part (4)) 3 total |  |  |
| - Algebraically show equality. Accuracy (no rounding) demonstrated | 2 |  |
| - Recognizes theorem from geometry | 1 |  |

Total out of 23 points

