Antiderivatives

Indefinite Integral – Classwork

Take a piece of notebook paper and cover up the paragraph under the chart below. Below, there are 5 terms. Write what you feel are the inverses of each of these terms.

Term	5	1/3	Boy	Dog	Hot dog
Its inverse					

As ridiculous as the last problem is (how can you have an inverse of a hot dog?) so are they all ridiculous? The problem is that all of these terms above are nouns. Inverses refer not just to opposite but to an opposite process. We take inverses of verbs, not nouns. Write the inverses of these processes.

Process	Sit down	Get dressed	Take a book home	Get wet	Go to sleep
Its inverse					

In mathematics, you have learned about operations on functions f. The inverse operation is denoted as f^{-1} (not to be confused with x^{-1}, the reciprocal of x) - remember x is a noun and f is the process which is a verb. Whenever you perform an operation and immediately perform its inverse, you will end up exactly where you started with. We say that $f^{-1}\left[f\left(x\right)\right]=x$ and also $f\left[f^{-1}\left(x\right)\right]=x$. Below write some mathematical functions and their inverses.

Function	Inverse

Function	Inverse

So, obviously since differentiation of functions is a process, we must have an inverse of that process. We call that process antidifferentiation. For instant, we know that the derivative of $y=x^3$ is $3x^2$. So it makes sense to say that the antiderivative $3x^2$ is x^3.

However, it is important to say <u>an</u> antiderivative of $3x^2$ rather than <u>the</u> antiderivative $3x^2$ for the simple reason that the derivative of $y=x^3$ is $3x^2$, but so is the derivative of $y=x^3+2$, $y=x^3-5$, and $y=x^3+6\pi$. There are an infinite number of functions whose derivative is $3x^2$. So when we go backwards to the antiderivative it is impossible to determine which function it came from. So to cover our bets, we say that the antiderivative $3x^2$ is x^3+C, where C represents a constant. We call C the constant of integration. It is important to attach the +C after every antiderivative. What you are doing is saying that the antiderivative is a family of functions rather than one specific function.

The process of taking antiderivatives is called integration, specifically, indefinite integration because of the constant of integration C. So we do not have to write the word antiderivative again, we us a symbol to represent an antiderivative. That symbol is called an integral sign which is written like $\int$. The way we write an integral is:

$\int f(x)\,dx = F(x) + C$. The dx tells you what the important variable is when you are integrating just as you need to know what the important variable is when you differentiate $\left(\frac{dy}{dt} \text{ as opposed to } \frac{dy}{dx}\right)$

So, since $\frac{d}{dx}(4x) = 4$, we will say that $\int 4\,dx = 4x + C$ and

since $\frac{d}{dx}(x^2 + 3x - 1) = 2x + 3$, we will say that $\int (2x+3)\,dx = x^2 + 3x + C$

Just as we have derivative rules, we have, we have a corresponding rule for integrals. Here are some basic integration rules.

Differentiation formula	integration formula
$\frac{d}{dx}[C] = 0$	$\int 0\,dx = C$
$\frac{d}{dx}[kx] = k$	$\int k\,dx = kx + C$
$\frac{d}{dx}[kf(x)] = kf'(x)$ a constant can be "factored out"	$\int kf(x)\,dx = k\int f(x)\,dx + C$ - factor out constant
$\frac{d}{dx}[f(x) \pm g(x)] = f'(x) \pm g'(x)$ The derivative of a sum is the sum of derivatives	$\int [f(x) \pm g(x)]\,dx = \int f(x)\,dx \pm \int g(x)\,dx + C$ integral of a sum is the sum of the integrals
$\frac{d}{dx}[x^n] = nx^{n-1}$ The power rule	$\int x^n\,dx = \frac{x^{n+1}}{n+1} + C$ The power rule reversed

Antiderivatives - Homework

Find the integral of each of the following:

1) $\int 7\ dx$

2) $\int x^5\ dx$

3) $\int x^{12}\ dx$

4) $\int \left(x^4 - x^2\right)dx$

5) $\int \left(t^3 + t + 1\right)dt$

6) $\int 3x^3\ dx$

7) $\int \left(2x^2 - 7x - 8\right)dx$

8) $\int \left(\frac{3}{4}x^5 + \frac{5}{3}x^2 - \frac{x}{2}\right)dx$

9) $\int \left(\pi x + \frac{1}{\pi}\right)dx$

10) $\int \frac{1}{x^2}\ dx$

11) $\int \left(\frac{4}{x^3} - \frac{5}{x^4}\right)dx$

12) $\int \sqrt{x}\ dx$

13) $\int \left(2\sqrt[3]{y} - 4\sqrt[4]{y}\right)dy$

14) $\int \left(\frac{1}{\sqrt{x}} - x^{\frac{2}{3}}\right)dx$

15) $\int \left(x^{\pi} + \sqrt{\pi}\right)dx$

In taking integrals, you may have to be clever. There are only a certain set of rules and if an integration problem doesn't fit one of the rules, you may have to change the expression so that it does. I call this a "bag of tricks".

Trick 1 – multiply out, then integrate

Trick 2 – Split into individual fractions, then integrate

16) $\int \left(2x - 3\right)^2\ dx$

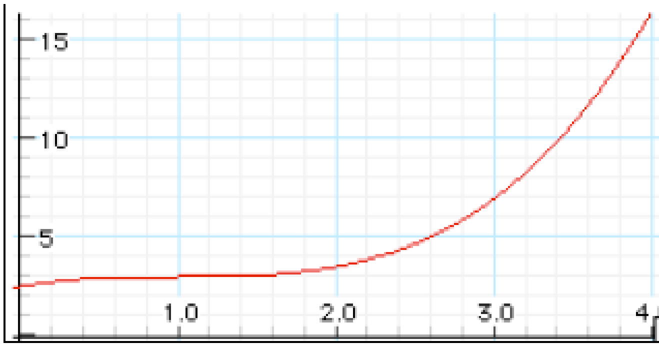
17) $\int \frac{x^2 + 3x + 1}{x^4}\ dx$

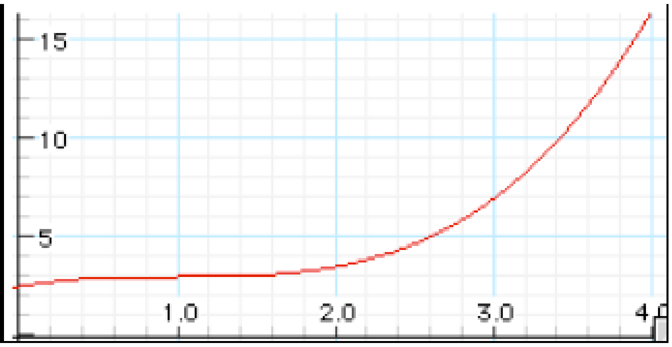
18) $\int \frac{\left(2x - 5\right)\left(3x + 2\right)}{\sqrt{x}}\ dx$

Area Under Curve - Classwork

One of the basic problems of calculus is to find the slope of the tangent line (i.e. the derivative) at any point on the curve. The other basic problem is to find the area under the curve, that is the area between the curve and the x-axis between any two values of x.

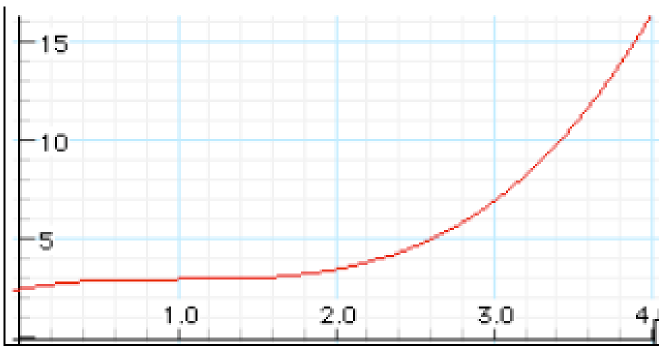
Below you are given a curve $y = f(x)$. Estimate what you think the area is. Then on the next three graphs, draw 2 rectangles, 4 rectangles, and 8 rectangles, total the areas of each and sum them for another estimate of the total area under the curve.

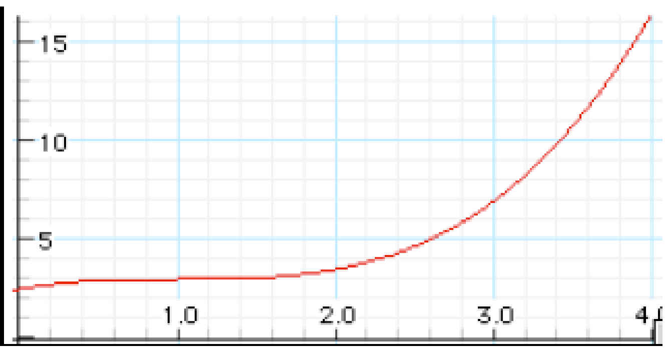




Estimate of area _______________

2 Rectangles: _______ + ________ = ______

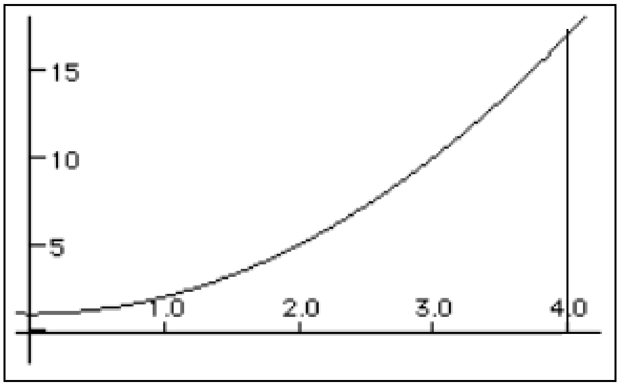




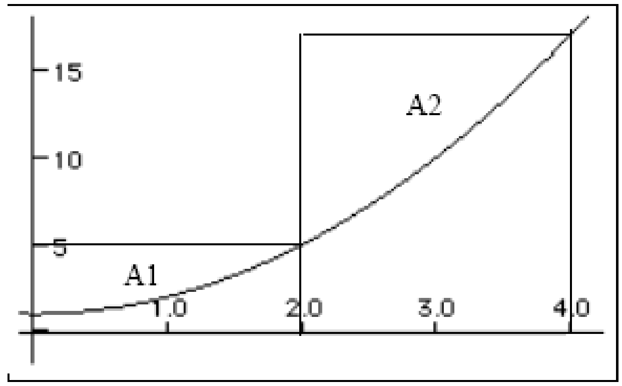
4 Rectangles: ____ + ____ + ____ + ____ = ______

8 Rectangles: ____ + ____ + ____ + ____ +
____ + ____ + ____ + ____ = ______

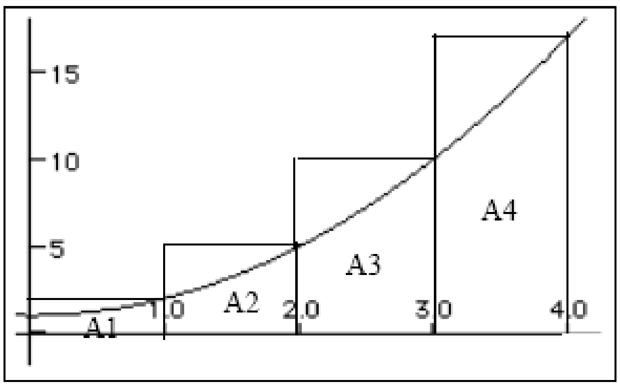
As you can see, as you use more rectangles, your estimate gets better but the amount of work you have to do increases. This is the idea we bring as we start our study of area. Now let's get more exact. Let's do the same thing with another curve, but this time you will be given the function: $f(x) = x^2 + 1$. Our problem is to estimate the area under the curve between $x = 0$ and $x = 4$ using "right" rectangles.



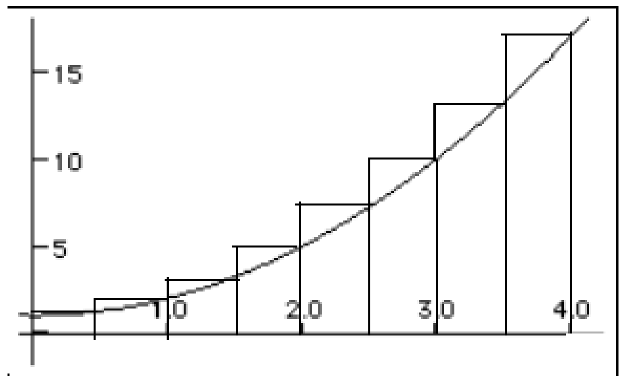
Estimate the area __________________



$$\text{Area } \approx A1 + A2 = b_1h_1 + b_2h_2$$
$$\text{Area} \approx b_1f(2) + b_2f(4) =$$



$$\text{Area } \approx A1 + A2 + A3 + A4$$
$$\text{Area} \approx b_1h_1 + b_2h_2 + b_3h_3 + b_4h_4$$
$$\text{Area} \approx b_1f(1) + b_2f(2) + b_3f(3) + b_4f(4) =$$



$$\text{Area } \approx A1 + A2 + A3 + A4 + A5 + A6 + A7 + A8$$
$$\text{Area} \approx b_1h_1 + b_2h_2 + b_3h_3 + b_4h_4 + b_5h_5 + b_6h_6 + b_7h_7 + b_8h_8$$
$$\text{Area} \approx b_1f(.5) + b_2f(1) + b_3f(1.5) + b_4f(2) +$$
$$b_5f(2.5) + b_6f(3) + b_7f(3.5) + b_8f(4) =$$

As you go through this process, several things should be apparent:

- Drawing the function is not really necessary.
- The more rectangles you create, the more work you have to do. It is just a lot of arithmetic.
- In each case, the base is same allowing you to factor it out. For instance in the last case above,

$$b_1f(.5) + b_2f(1) + b_3f(1.5) + b_4f(2) + b_5f(2.5) + b_6f(3) + b_7f(3.5) + b_8f(4) =$$
$$b\left[f(.5) + f(1) + f(1.5) + f(2) + f(2.5) + f(3) + f(3.5) + f(4)\right]$$

- The more rectangles you create, the more accurate the area should be. So it should be apparent that

$$\text{True Area } = \lim_{n\to\infty} \sum_{i=1}^{n} A_i$$

Note: Look ahead to page 8 and find the program you will get tomorrow. It can help with the homework.

Approximating Area Under the Curve--Homework

For the following, sketch the graph, the designated rectangles, then approximate the area.

Examples) Find the area under the following functions using the indicated number of rectangles:

1) $f(x)=3x+1$ on $[1,5]$

   a) 4 b) 8

2) $f(x)=x^2+3$ on $[2,5]$

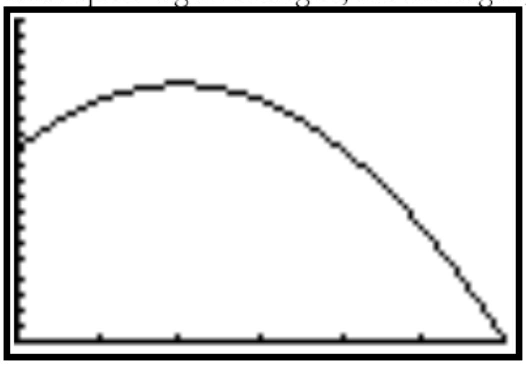
   a) 3 b) 6

3) $f(x)=x^2-3x-2$ on $[4,6]$

   a) 8 b) 16

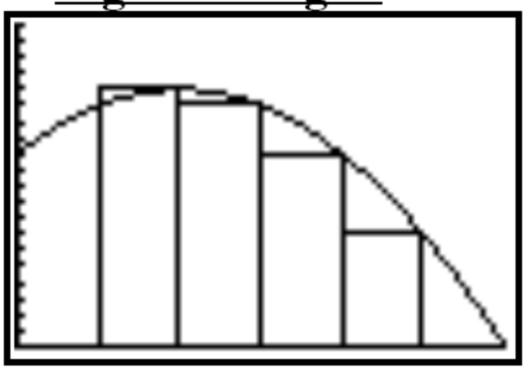
Riemann Sums - Classwork

As a common example, this worksheet will use this problem. Find the area under the function $f(x)$ given in the picture below from $x = 1$ to $x = 5$. What we are looking for is the picture on the right. We will look at 4 techniques: right rectangles, left rectangles, midpoint rectangles and trapezoids.



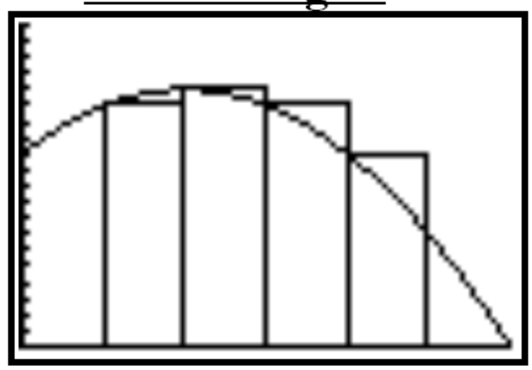
First some common statements. We will use 4 rectangles or trapezoids in this worksheet but you are expected to learn the technique for any number of rectangles or trapezoids. Obviously, if we wish 4 rectangles and the values of x run from 1 to 5, the base of each rectangle is 1. Here are the 4 pictures of what we are looking for.

Right rectangles



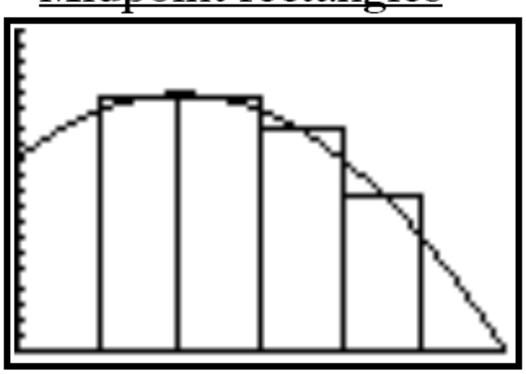
The height of the rectangle is on the right side.
This will underestimate the area (this case only).

Left rectangles



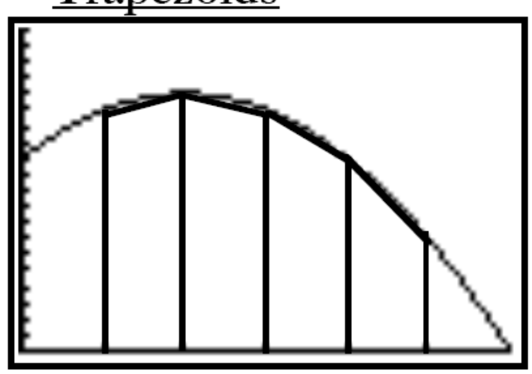
The height of the rectangle is on the left side.
This will overrestimate the area (this case only).

Midpoint rectangles



The height of the rectangle is in the middle.
This ends up both over and underestimating the area.

Trapezoids



The vertical lines represent the bases of the trapezoids
The result is a very good approximation to the area.

When we divide our picture into 4 rectangles, we have to find the base of each rectangle. In this case, since we were interested in doing this from $x = 1$ to $x = 5$, and there were 4 rectangles, we lucked out because the base of each rectangle is 1. That won't always happen. In general, let's call the base $= b$.

Now let us define $x_0, x_1, x_2, \ldots x_n$ as the places on the x-axis where we will build our heights, where n represents the number of rectangles (or trapezoids). In this case,

$$x_0 = 1,\ x_1 = 2,\ x_2 = 3,\ x_3 = 4,\ x_4 = 5.$$

In the case of **right** rectangles, the area will be:

$$A \approx bh_1 + bh_2 + bh_3 + bh_4$$
$$A \approx b(h_1 + h_2 + h_3 + h_4)$$

In the case of **left** rectangles, the area will be:

$$A \approx bh_0 + bh_1 + bh_2 + bh_3$$
$$A \approx b(h_0 + h_1 + h_2 + h_3)$$

but since $h_i = f(x_i)$, we can say

$$A \approx b\left(f(x_1) + f(x_2) + f(x_3) + f(x_4)\right) \qquad A \approx b\left(f(x_0) + f(x_1) + f(x_2) + f(x_3)\right)$$

so, in the specific case above

$$A \approx 1\left(f(2) + f(3) + f(4) + f(5)\right) \qquad A \approx 1\left(f(1) + f(2) + f(3) + f(4)\right)$$

so in general:

$$A \approx b\sum_{i=1}^{n} f(x_i) \qquad A \approx b\sum_{i=0}^{n-1} f(x_i)$$

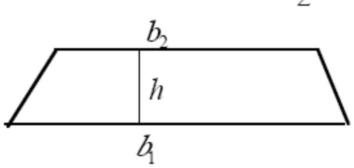
These are called Riemann Sums.

In the case of **midpoint rectangles**, you have to find the midpoint between your $x_0, x_1, x_2, \ldots, x_n$
The midpoint between any two x values is their sum divided by 2, so you will use:

$$A \approx b\left[f\left(\frac{(x_0 + x_1)}{2}\right) + f\left(\frac{(x_1 + x_2)}{2}\right) + f\left(\frac{(x_2 + x_3)}{2}\right) + f\left(\frac{(x_3 + x_4)}{2}\right)\right]$$

In our case, $A \approx 1\left[f\left(\frac{(1+2)}{2}\right) + f\left(\frac{(2+3)}{2}\right) + f\left(\frac{(3+4)}{2}\right) + f\left(\frac{(4+5)}{2}\right)\right]$ or $A \approx 1\left(f(1.5) + f(2.5) + f(3.5) + f(4.5)\right)$

For **trapezoids**, remember that area $= \frac{1}{2}$ • height • $(b_1 + b_2)$. That is when the trapezoid looks like this:



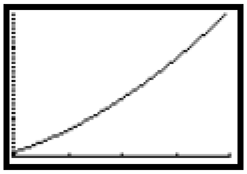
Since our traezoids are on their sides, we will say
$$A = \frac{1}{2} \cdot \text{base} \cdot (h_1 + h_2)$$

So, the total area $A \approx \frac{1}{2}b\left[\left(f(x_0) + f(x_1)\right) + \left(f(x_1) + f(x_2)\right) + \left(f(x_2) + f(x_3)\right) + \left(f(x_3) + f(x_4)\right)\right]$

or, in our case $A \approx \frac{1}{2}b\left[f(x_0) + 2f(x_1) + 2f(x_2) + 2f(x_3) + f(x_4)\right]$ $\quad A \approx \frac{1}{2}b\left[f(1) + 2f(2) + 2f(3) + 2f(4) + f(5)\right]$

In general, the **trapezoidal rule**: $A \approx \frac{1}{2}b\left[f(x_0) + 2f(x_1) + 2f(x_2) + 2f(x_3) + \ldots + 2f(x_{n-1}) + f(x_n)\right]$

Let's try one: Let $f(x) = x^2 - 3$. We want to find the area under the curve using 8 rectangles/trapezoids from $x = 2$ to $x = 6$. First, let's draw it. Note that the curve is completely above the axis. If it dips below, the method changes slightly.



The drawing of the curve is helpful, but not necessary.

Since there are 8 rectangles, and we are finding the area between $x = 2$ and $x = 6$, the base is __________

Let's complete the chart:

i	x_i	$f(x_i)$
0		
1		
2		
3		
4		
5		
6		
7		
8		

So, the right rectangle formula gives ____

the left rectangle formula give ____

the trapezoid formula gives ____

Note that the chart will not give you the midpoint formula. Let's do it here:

____

To simplify the work, get the Riemann program from your teacher.

Make notes about the menu functions here:

Menu item	What it does
1. Set Parameters	
2. Left Sum	
3. Right Sum	
4. Midpoint Sum	
5. Trapezoidal Sum	
6. Def. Integral	
7. Quit	

Riemann Sums - Homework

For each problem, approximate the area under the given function using the specified number of rectangles/ trapezoids. You are to do all 4 methods to approximate the areas.

#	Function	Interval	Number	Left Rectangles	Right Rectangles	Midpoint Rectangles	Trapezoids
1	$f(x) = x^2 - 3x + 4$	[1,4]	6				
2	$f(x) = \sqrt{x}$	[2,6]	8				
3	$f(x) = 2^x$	[0,1]	5				
4	$f(x) = \sin x$	$[0,\pi]$	8				

Sketch each. Show left rectangles:

1.

2.

3.

4.

Answers are below:

#	Left Rectangles	Right Rectangles	Midpoint Rectangles	Trapezoids
1	9.125	12.125	10.438	10.625
2	7.650	8.168	7.914	7.909
3	1.345	1.545	1.442	1.445
4	1.974	1.974	2.013	1.974

Review Day 1-3

Find the antiderivative, f(x)

1. $f'(x) = 3x^2 + 10x + 4$

2. $f'(x) = 20x^3 + 12x - 13$

3. $f'(x) = \frac{-2}{x^3}$

4. $f'(x) = \frac{5}{\sqrt[3]{x}}$

5. $f'(x) = \frac{-1}{x^2}$

6. $f'(x) = \frac{7}{2\sqrt{x}}$

Estimate the area under the curve on the interval using 5 right rectangles .

7. $f'(x) = e^x$ $[0,2]$

8. $f'(x) = \cos x$ $[0, \frac{\pi}{2}]$

9. Use the Riemann sum principles to approximate the area under $\dfrac{3}{2\sqrt{1+3x}}$ on the interval [0,5]

a. using 10

i. right rectangles

ii. left rectangles

iii. trapezoids

iv. midpoints

b. Which should best approximate the area?

c. Sketch the curve at the right and determine which should

->overestimate the area

->underestimate the area

Finding the Exact Area Under a Curve - Classwork

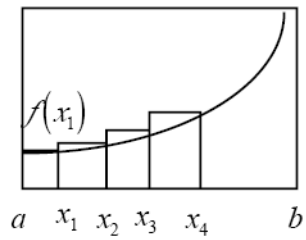
Now that we have used a finite number of right, left, and mid rectangles as well as trapezoids to approximate the area under a curve, we now extend the concept to finding the exact area under the curve by looking at n rectangles and calculating the limit of the sum of the areas of those rectangles as n approaches infinity.

In the method shown below, we will assume right rectangles. Left rectangles are possible as well but since either method will give the same answer, we will just settle on one method. This and the next page will be set up so that the general method and description will be on the left side of the page, and an example (specific problem) will be on the right side.

General Problem: Find the exact area between a curve $y = f(x)$, the x-axis, and $x = a$ and $x = b$. We assume the graph will be above the x-axis.

• Make a simple sketch of the function

Specific Problem: Find the exact area between the curve $f(x) = x^2 + 1$, the x-axis, and $x = 1$ and $x = 3$



To carry out this process, go to http://mathworld.wolfram.com/RiemannSum.html

1. Enter the function.
2. Enter the limits for x.
3. Enter the number of rectangles.
4. Increase the number of rectangles until the estimated area is within 0.01 of the actual area.
5. Record approximate area and the number of rectangles used.
6. Record actual area as: "Actual area = $\lim_{n\to\infty}\sum_{i=1}^{n} A_i$ =______

Finding the Exact Area Under a Curve - Homework

Find the area between the following curves, the x-axis, between the two given x-values:

1. $f(x) = 4x - 2$, $x = 1$ and $x = 4$ Answer: 24

2. $f(x) = x^2 + x$, $x = 2$ and $x = 4$ Answer: $\frac{74}{3}$

3. $f(x) = 2x^2 - 3x + 1$, $x = 1$ and $x = 2$ Answer: $\frac{7}{6}$

4. $f(x) = 9 - x^2$ and the x-intercepts Answer: 36

5. $f(x) = x^3$, $x = 0$ and $x = 4$ Answer: 64

Definite Integrals – Fundamental Theorem of Calculus

Technology: Your TI-84 calculators allow you to find the value of a definite integral. The command is $\boxed{\text{FnInt}}$ and is located as #9 in the $\boxed{\text{Math}}$ menu. The syntax of the statement is FnInt(expression in X, X, lower, upper). For instance, example # 1 above -- $\int_1^2 x^2\,dx$ would be expressed to the calculator as FnInt(X^2, X,1,2) yielding 2.333. You can also put your expression in Y1 and your statement would be FnInt(Y1, X,1,2). The calculator is finding the integral but not by the Fundamental Theorem of Calculus. It is merely performing the summation of rectangles or trapezoids many many times.

Problems 1-3, use rules learned in class—Take notes! Problems 4-8, use calculator.

1. $\int_1^2 2x - x^2 dx$

2. $\int_1^4 \frac{1}{2\sqrt{x}} dx$

3. $\int_1^3 \frac{1}{x^2} dx$

4. $\int_e^{e^3} \frac{1}{x} dx$

5. $\int_0^{\pi} \sin x dx$

6. $\int_{\pi/6}^{\pi/3} \sec^2 x dx$

7. $\int_1^5 \frac{3}{2\sqrt{1+3x}} dx$

8. $\int_1^2 2xe^{x^2+3} dx$ let $u = x^2 + 3$

Note: Calculator only on 11-15

The Fundamental Theorem of Calculus - Homework

Find the value of the definite integrals below. Confirm using your calculator.

1. $\int_0^1 3x\,dx$

2. $\int_{-2}^{3} (x-5)\,dx$

3. $\int_{-1}^{4} (x^2+2x-1)\,dx$

4. $\int_0^2 (2x-5)^2\,dx$

5. $\int_2^3 \left(\frac{4}{x^2}+1\right)dx$

6. $\int_{-2}^{-1} \left(x-\frac{1}{x^2}\right)dx$

7. $\int_1^9 \frac{x-2}{\sqrt{x}}\,dx$

8. $\int_{-2}^{2} \sqrt[3]{x}\,dx$

9. $\int_0^1 \left(t^{2/3}-t^{1/3}\right)dt$

10. $\int_0^3 |x-2|\,dx$

11. $\int_{-\pi/2}^{\pi/2} \cos x\,dx$

12. $\int_0^{\pi} (2x-\sin x)\,dx$

13. $\int_0^{\pi/2} (3\sin x-2\cos x)\,dx$

14. $\int_0^{\pi/4} (x-\sec^2 x)\,dx$

15. $\int_0^{\pi/3} \sec\theta\tan\theta\,d\theta$

Individual Test Review

1. Find the antidervatives:

a. $\int (3x^4 - 7x^2 + 6x - 9)\, dx$

b. $\int -\frac{2}{x^5}\, dx$

c. $\int \frac{3}{4\sqrt{x}}\, dx$

d. $\int \left(\frac{1}{\sqrt{x}} + \sqrt{x} \right) dx$

2. Find the area under the curve:

i. Sketch and label the curve on the designated interval.

ii. Approximately using 5 right rectangles, 5 left rectangles and 5 trapezoids.

iii. Determine, if possible, which over and underestimate.

iv. Using the website in the notes, increase the number of rectangles until the area is unchanged in the second decimal place. Record n.

v. Compare this result to TI result using fnInt

a. $\int_0^{2\pi} 2\left|\cos x\right| dx$

b. $\int_{-1}^{2} (e^x - 2x) dx$

3. Find the definite integrals below both with rules and calculator:

a. $\int_{3}^{8} 2x\, dx$

b. $\int_{0}^{4} 20\, dx$

c. $\int_{1}^{16} \frac{3}{2\sqrt{x}}\, dx$

d. $\int_{1}^{5} \frac{1}{x^4}\, dx$

e. $\int_{2}^{6} (3-4x^3)\, dx$

f. $\int_{0}^{3} (3x^5+12x+2)\, dx$

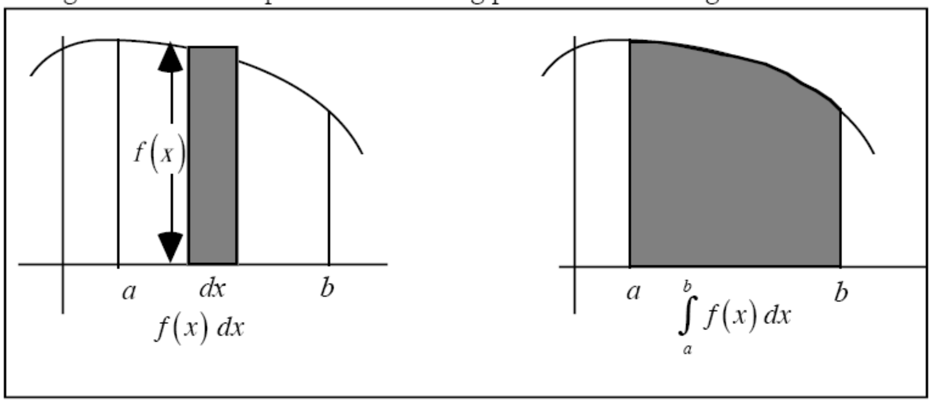
i. $\int_{0}^{4} \frac{3\sqrt{x}}{4}\, dx$

j. $\int_{2}^{4} \frac{8}{x^3}\, dx$

Answers: (1a) $\frac{3}{5}x^5 - \frac{7}{3}x^3 + 3x^2 - 9x + C$ (1b) $\frac{1}{2x^4} + C$ (1c) $\frac{3\sqrt{x}}{2} + C$ (1d) $2\sqrt{x} + \frac{2\sqrt{x^3}}{3} + C$ (2a) (ii) 8.1331, 8.1331, 8.1331 (iv) 8 (b) (ii) 4.537, 3.924, 4.231 (iv) 4.021 (3a) 55 (b) 80 (3c) 9 (3d) .331 (3e) -1268 (3f) 424.5 (3i) 4 (3j) 3/4

The Definite Integral as Area - Classwork

Instead of using the expression "the area under the curve $f(x)$ between $x = a$ and $x = b$, we will now denote a shorthand to represent the same thing. We will use what is called "a definite integral." The definite integral sign is the same as the indefinite integral sign ($\int$) but will contain two limits of integration. The form is as follows: $\int_a^b f(x)\,dx$. While this does not seems to make much sense, there is a reason for it. The $f(x)$ represents the height of any one rectangle while the dx represents the width of any one rectangle. So $f(x)\,dx$ means the area of any one rectangle. The integral represents the sum of an infinite number of these rectangles. The a represents the starting place for these rectangles while the b represents the ending place for these integrals.



The area of one rectangle = $f(x)\,dx$

The sum of an infinite number of rectangle areas. Each rectangle is infinitely thin.

When $a < b$, we are determining the area under the curve from left to right. In that case, our dx is a positive number. If $f(x)$ is above the axis, then $\int_a^b f(x)\,dx$ will be a positive number.

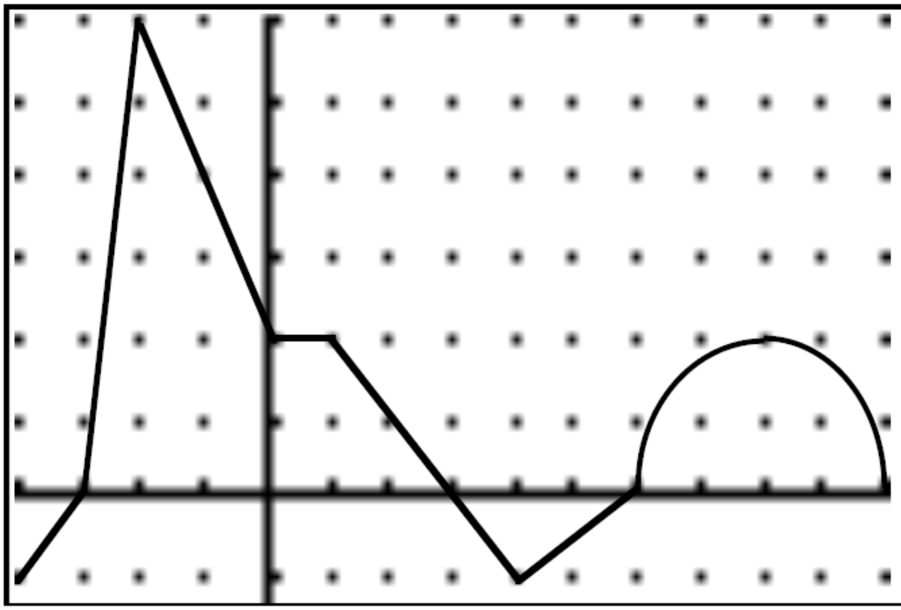
When $b < a$, we are determining the area under the curve from right to left. In that case, our dx is a negative number. If $f(x)$ is above the axis, then $\int_a^b f(x)\,dx$ will be a negative number. This can be summarized below:

	$f(x) > 0$ (curve above axis)	$f(x) < 0$ (curve below axis)
$dx > 0$ (left to right) ($a < b$)	$\int_a^b f(x)\,dx > 0$ (Area positive)	$\int_a^b f(x)\,dx < 0$ (Area negative)
$dx < 0$ (right to left) ($b < a$)	$\int_a^b f(x)\,dx < 0$ (Area negative)	$\int_a^b f(x)\,dx > 0$ (Area positive)

Furthermore, there are three more rules which will make sense to you:

1. $\int_a^a f(x)\,dx$ - If we start at a and end at a, there is no area.
2. $\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$ - From a to b gives an area. From b to a gives the negative of this area.
3. $\int_a^b f(x)\,dx + \int_b^c f(x)\,dx = \int_a^c f(x)\,dx$ - Total the area from a to b, add area from b to c = the area from a to c.

Example) Below you are given the graph of $f(x)$ formed by lines and a semi-circle. Find the definite integrals.



This is the graph of $f(x)$

1. $\int_{4}^{4} f(t)\,dt$
2. $\int_{0}^{1} f(t)\,dt$
3. $\int_{1}^{3} f(t)\,dt$
4. $\int_{0}^{3} f(t)\,dt$
5. $\int_{3}^{6} f(t)\,dt$
6. $\int_{6}^{3} f(t)\,dt$
7. $\int_{0}^{6} f(t)\,dt$
8. $\int_{6}^{10} f(t)\,dt$
9. $\int_{10}^{6} f(t)\,dt$
10. $\int_{0}^{10} f(t)\,dt$
11. $\int_{10}^{0} f(t)\,dt$
12. $\int_{-1}^{0} f(t)\,dt$
13. $\int_{-3}^{0} f(t)\,dt$
14. $\int_{0}^{-3} f(t)\,dt$
15. $\int_{-4}^{-3} f(t)\,dt$
16. $\int_{-4}^{0} f(t)\,dt$
17. $\int_{-4}^{10} f(t)\,dt$
18. $\left|\int_{0}^{10} f(t)\,dt\right|$
19. $\int_{0}^{10} \left|f(t)\right|\,dt$
20. $\left|\int_{-4}^{10} f(t)\,dt\right|$
21. $\int_{10}^{-4} \left|\,2f(t)\,\right|\,dt$

Interpretation of Area

1. A car comes to a stop 5 seconds after the driver slams on the brakes. While the brakes are on, the following velocities are recorded. Estimate the total distance the car took to stop.

Time since brakes applied (sec)	0	1	2	3	4	5
Velocity (ft/sec)	88	60	40	25	10	0

2. You jump out of an airplane. Before your parachute opens, you fall faster and faster. Your acceleration decreases as you fall because of air resistance. The table below gives your acceleration a (in m/sec^2) after t seconds. Estimate the velocity after 5 seconds.

t	0	1	2	3	4	5
a	9.81	8.03	6.53	5.38	4.41	3.61

3. Cedarbrook golf course is constructing a new green. To estimate the area of the green, the caretaker draws parallel lines 10 feet apart and then measures the width of the green along that line. Determine how many square feet of grass sod that must be purchased to cover the green if

a) The caretaker is lazy and uses midpoint rectangles to calculate the area.
b) The caretaker uses left rectangles to calculate the area.
c) The caretaker uses right rectangles to calculate the area.
d) The caretaker uses trapezoids to calculate the area.

Width in feet	0	28	50	62	60	55	51	30	3

Interpretation of Area Homework (Complete on separate paper)

5. Roger decides to run a marathon. Roger's friend Jeff rides behind him on a bicycle and clocks his pace every 15 minutes. Roger starts out strong, but after an hour and a half he is so exhausted that he has to stop. The data Jeff collected is summarized below. Assuming that Roger's speed is always decreasing, estimate the distance that Roger ran in a) the first half hour and b) the entire race. (Trapezoids)

Time spent running (min)	0	15	30	45	60	75	90
Speed (mph)	12	11	10	10	8	7	0

6. Coal gas is produced at a gasworks. Pollutants in the air are removed by scrubbbers, which become less and less efficient as time goes on. Measurements are made at the start of each month (although some months were neglected) showing the rate at which pollutants in the gas are as follows. (trapezoids)

Time (months)	0	1	3	4	6	7	9
Rate pollutants are escaping (tons/month)	5	7	8	10	13	16	20

7. For $0 \le t \le 1$, a bug is crawling at a velocity v, determined by the formula $v = \dfrac{1}{1+t}$, where t is in hours and v is in meters/hr. Find the distance that the bug crawls during this hour using 10 minute increments.

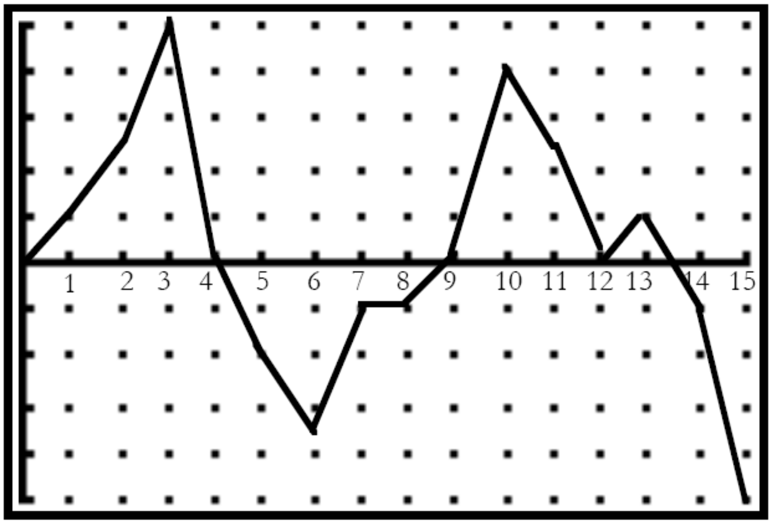
6. An object has zero initial velocity and a constant acceleration of 32ft/sec^2. Complete the chart to find the velocity at these specified times. Then determine the distance traveled in 4 seconds.

t (sec)	0	.5	1	1.5	2	2.5	3	3.5	4
v (ft/sec)									

The Accumulation Function - Life Application

You have bought 100 shares of XYZ stock and decide to keep it for 15 days. Below is a graph that represents the change of the price of the your stock on each day. For instance, on the end of day 1, the stock has increased by $1 a share. At the end of day 4, it has not changed. At the end of day 6, it has gone down by $3.50 a share.

We will call the graph you see below $f(t)$. It is obviously a function of time. Remember that the function represents the *change* in the value of your stock, **not** the value of the stock.



Let $F(x) = \int_0^x f(t)\,dt$. Answer the following questions:

1. Complete the chart below.

x	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
$f(x)$																
$F(x)$																

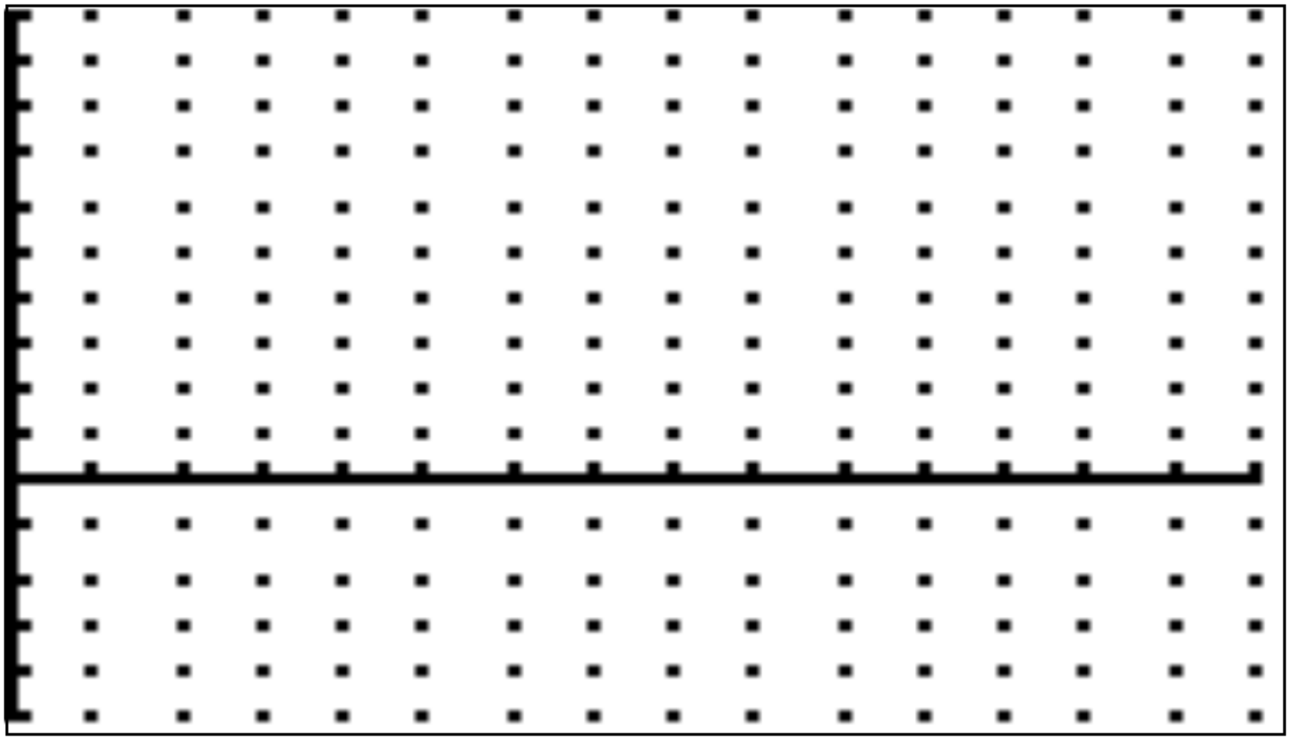
2. What is the real life meaning of $F(x)$?

3. For what intervals is $F(x)$ increasing and decreasing. Justify your answer.

4. In the interval [0, 15], find the minimum value of $F(x)$ and the day it is reached. Justify your answer.

5. In the interval [0, 15], find the maximum value of $F(x)$ and the day it is reached. Justify your answer.

7. Below, sketch $F(x)$.



8) Based on your findings, find:

a) how much money you made(lost) on XYZ stock in that 15 day time period.

b) the day that the stock's value had the biggest rise

c) the days between which the stock's value had the steepest rise (not the same question)

d) the day that the stock's value had the biggest decline

e) the days between which the stock's value had the steepest decline (not the same question)

f) the day you wished you sold your stock

g) the day you are glad you didn't sell your stock

Accumulation Functions Homework

1. During a recent snowfall, several students monitored the accumulation of snow on the flat roof of their school. The table below records the data they collected for the 12-hour period of the snowfall.

Number of Hours	Rate of Snowfall (inches/hour)
0	0
2	1.5
3	2.1
4.5	2.4
6.5	2.8
8	2.2
10.5	1.8
12	1.6

a) Use a right hand sum to approximate the total depth of snow in the 12-hour period.

b) Using the right-hand sum, estimate the average rate of snowfall in the 12-hour period.

2. A car slows down as it approaches a red light at an intersection. When the light turns green, the velocity of the car increases as shown in the table.

Time t (seconds)	Velocity v (feet/second)
0	8
2	14
4	22
6	30
8	40
10	45

a) Find the average rate of change of the velocity v on the interval [0, 10].

b) Approximate the distance travelled in the first ten seconds using trapezoids and 5 equal subintervals.

c) Approximate the acceleration of the car at $t = 6$.

More Accumulation Functions

Applications can become increasingly complex. In fact, most of the mathematical models that have been developed in math and science classes were derived through differential equations, which require the integral for solutions. The most important aspect of the following applications is to learn to read through the language and to understand which calculus technique needs to be applied to answer the question.

Example: Sirius Radio launched an aggressive advertising campaign to attract viewers in 2005. They estimated that the number of listeners, *L(t)*, would grow at the rate of $L'(t) = 18,000t^{\frac{1}{2}}$ listeners per year. In 1997, Sirius received approval from the FCC to broadcast. What is the net change in listeners from 1997 to 2009?

Solution: We need to use the function which describes the rate of change of the listeners to answer this question. Create a table for *L'(t)* and *L(t), (*remember, $L(t) = L(0) + \int_0^t L'(x)\,dx$) for values of t which correspond to the years, 1997, 2000, 2003, 2006 and 2009. Include units in the column headings:

Express an answer to the question:

If there were 1000 listeners in 1997, how would this change the answer to the question?

Project the number of listeners in 2015 based on this information. Assume that the estimate was accurate for 2009, but that, you find that your estimate was 50% below the actual number in 2015. Explain what may have happened and how we should change our function, L'(t), and our method to find the projected number of listeners.

More Accumulation Functions --Homework

1. The daily circulation of a local newspaper has been affected by the influx of media available via the Internet. Its circulation is decreasing at the rate of $C'(t) = -0.6t^{\frac{1}{3}}$ copies per day, where t is the number of days since 2000. The daily circulation was 640,000 copies per day in 2000. What is the circulation today?

2. The maintenance costs for an apartment building generally increase as the building gets older. From past records, a managerial service determined that the rate of increase in maintenance costs (in dollars per year) for a particular building are given by , $M'(x) = 90x^2 + 5000$, where x is the age of the building in years and *M(x)* is the total cost of maintenance for x years.

(a) Write an integral which gives the total maintenance costs from 2 to 7 years after the apartment was built.

(b) Evaluate the integral, and explain its meaning.

3.. Using data from the first three years of production, the management of a oil company estimates that gas will be pumped at the rate given by $R(t) = \frac{100}{t+1} + 5,\ 0 \le t \le 20$, where *R(t)* is the rate of production (in thousands of barrels per year) t years after pumping began.

(a) Approximately how many barrels of oil will the field produce during the first 10 years of production?

(b) Approximately how many barrels of oil will the field produce from the end of the 10^{th} year to the end of the 20^{th} year?

(c) If 10,000 barrels were produced in the 1^{st} year, how many barrels were produced in the 5^{th} year?

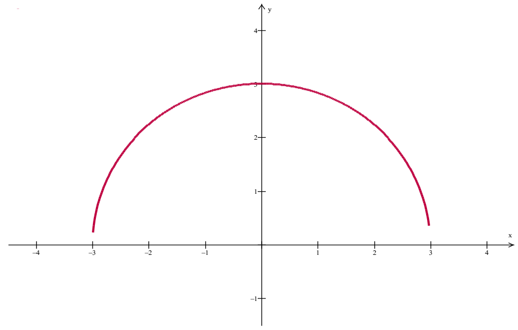
4. A company manufactures *x* refrigerators per month. The monthly change in profit is given by: $P'(x) = 165 - 0.1x,\ 0 \le x \le 4000$. (dollars per refrigerator). The company is currently manufacturing 1,500 sets per month, but is planning to increase its production.

Find the total change in monthly profit if monthly production is increased to 1,600 sets.

Partner Test Review

Geometric Areas

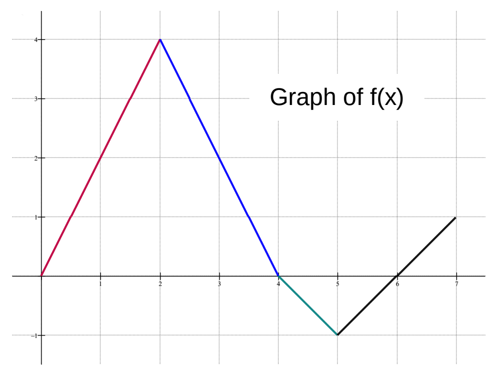
1. Evaluate the integrals below based on the graph of $y = \sqrt{9 - x^2}$ shown.



a. $\int_{-3}^{3} \sqrt{9 - x^2}\, dx$

b. $\int_{0}^{3} \sqrt{9 - x^2}\, dx$

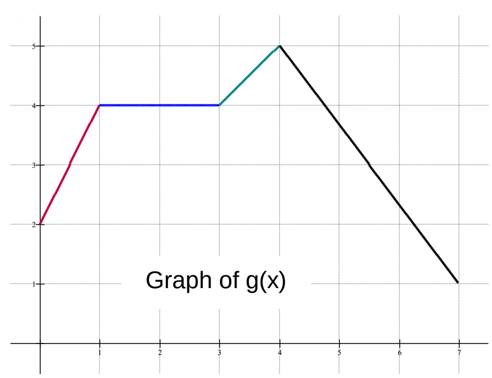
2. Evaluate the following integrals based on the graph of f(x).



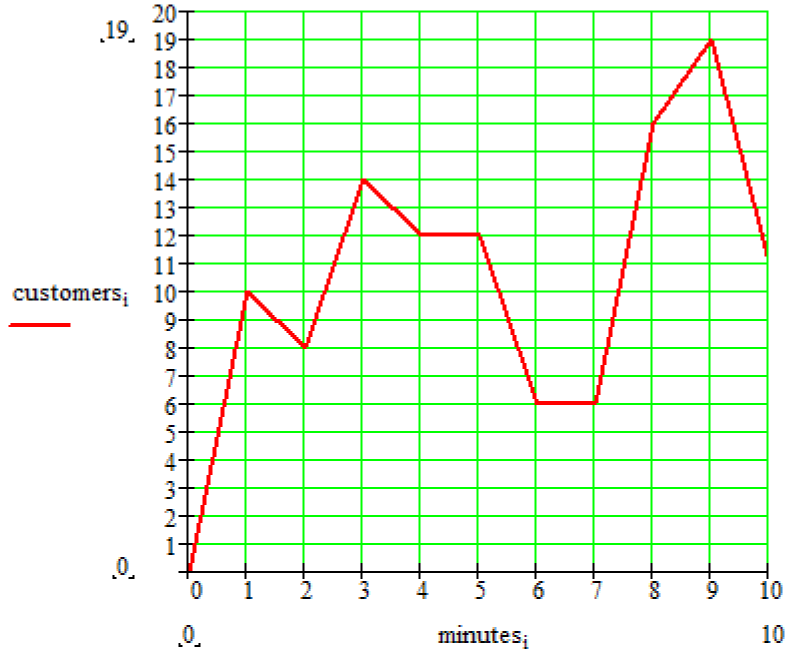
a. $\int_{0}^{4} f(x)\, dx$

b. $\int_{3}^{7} f(x)\, dx$

3. $\int_{0}^{7} g(x)\, dx$



4. Ms. Doe is a greeter for Walmarts. One of her duties is counting the number of customers entering the store each minute. At the right are the results for a ten-minute interval. The number of customers per minute is modeled by the function $f(t)$.



a. Using left-side rectangles, determine how many customers she observed between t = 3 and t = 7?

b. Using right-side rectangles, determine how many customers she observed between t = 3 and t = 7?

c. Evaluate $\int_3^7 f(t)\,dt$ and explain its meaning using units appropriate to the problem.

d. Explain the discrepancy between your answers in (a), (b), and (c). Which is accurate?

5. A horse trots along a straight, flat road for 30 seconds. Its velocity in feet per second during the time interval $0 \le t \le 30$ seconds is shown in the table

t (sec)	0	5	10	15	20	25	30
v(t) (ft/sec)	0	15	20	24	22	20	16

a. Using right-side rectangles, approximate the value of $\int_0^{30} v(t)\, dt$.

b. Using left-side rectangles, approximate the value of $\int_0^{30} v(t)\, dt$.

c. Using trapezoids, approximate the value of $\int_0^{30} v(t)\, dt$.

d. Using the trapezoidal sum, estimate the average velocity of the horse for the 30 minutes shown.

6. Jeremy likes to jog on a mountain trail near his home. His velocity in feet/minute is determined by the difficulty of the terrain and is given by the differentiable function *V* of time *t*. The table below shows his velocity measured every 2 minutes for 20 minutes.

t (minutes)	0	2	4	6	8	10	12	14	16	18	20
v(t) meters per minute	0	708	936	594	681	765	864	690	557	633	603

1. What does the definite integral $\int_0^{20} V(x)dx$ tell you about Jeremy' s run?

2. Use the table above to get an approximation $\int_0^{20} V(x)dx$ of .

a. Show the correct base for each rectangle or trapezoid.

b. Show how you arrive at the height for each rectangle or trapezoid.

c. Show how to get a good approximation for $\int_0^{20} V(x)dx$ using the wisest strategy for rectangle/trapezoids.

2. Use a midpoint Riemann sum with 5 subdivisions to approximate $\frac{1}{5{,}280}\int_0^{20} V(t)\,dt$. Using correct units, explain the meaning of your answer.

Answers:

(1a) $\frac{9\pi}{2}$ ft/sec (b) $\frac{9\pi}{4}$ (2a) 8 (b) ½ (3) 24.5 (4a) 44 (b) 36 (c) 40 (d) (5a) 35,100 ft (b) 30,300 ft (c) 32,700 ft (d) 18.167

(6) See EXCEL file “Jeremy” on Moodle